Chapter 8
Design of Walls

8.0 NOTATION

a_u	Deflection due to slenderness of wall
a_{cj}	Distances from compression face to centroid of layers of concrete in compression
a_{tj}	Distances from compression face to centroid of layers of tensile reinforcement
A	Area bounded by median line of wall in closed cell
A_c	Net area of concrete in a section of wall
A_C	Centroid of compression in a wall section
A_h	Area of steel in shear reinforcement placed horizontally in in-plane direction
A_T	Centroid of tensile steel in a wall section
A_v	Area of steel in shear reinforcement placed vertically
A_{sc}	Area of steel in compression in a section of wall
A_{si}	Total area of steel in tension in a wall section for in-plane bending
A_{so}	Total area of steel in tension in a wall section for out-of-plane bending
A_{stj}	Layers of tensile steel reinforcement in wall for stress analysis
A'_{stj}	Layers of compressive steel reinforcement in wall for stress analysis
b	Actual width of flange of a shear wall
b	Unit width of wall for out-of-plane bending
b_e	Effective width of flange of a shear wall
B	Plan length of wall for the computation of moment of inertia
c	Coefficient to determine torsional stiffness of a rectangular section
C	Torsional stiffness of a rectangular section
d	Effective depth from compression face to centroid of tensile steel
d_i	Effective depth of tensile steel in wall for in-plane bending
d_o	Effective depth of tensile steel in wall for out-of-plane bending
e	Eccentricity of load on wall section for in-plane bending
e_a	Slenderness coefficient of slender braced plain wall
e_x	Resultant eccentricity of all loads at right angles to plane of wall
$e_{x,1}$	Resultant eccentricity of loads at top of wall
$e_{x,2}$	Resultant eccentricity of loads at bottom of wall
E	Modulus of elasticity
f_c	Stress in concrete compression
f_y	Characteristic yield strength of reinforcement
f_{cu}	Characteristic cube strength of concrete at 28 days
f_{st}	Tensile stress in steel reinforcement

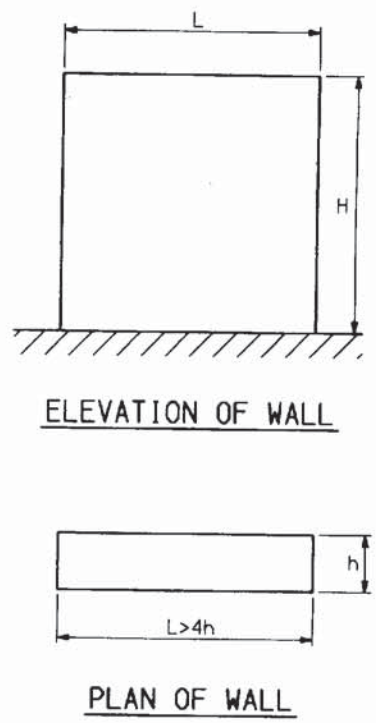
G	Modulus of rigidity
h	Thickness of wall
h_f	Thickness of flange of a shear wall section
h_w	Thickness of web of a shear wall section
H	Height of wall
H_e	Effective height of wall
H_o	Clear height of wall
J	Torsional stiffness of a closed cell structure
K	Factor to determine additional moment due to slenderness
L	Length of wall in in-plane direction
m	Modular ratio E_s/E_c
M	Applied bending moment on a concrete section
M'	Modified applied moment to account for axial load
M_I	In-plane applied bending moment in a wall section
M_{OH}	Out-of-plane bending moment in wall about horizontal plane
M_{OV}	Out-of-plane bending moment in wall about vertical plane
M_{add}	Additional bending moment in out-of-plane direction due to slenderness
n_w	Total design ultimate axial load on a wall
N	Axial load
N_{OV}	In-plane axial load due to out-of-plane loading on wall panel
p_i	Percentage of tensile steel for in-plane bending of wall
p_o	Percentage of tensile steel for out-of-plane bending of wall
q	Shear flow in components of a closed cell (kN/m)
Q_I	In-plane shear flow due to torsion in a closed cell
R	Restraint factor
s	Median length of wall
S_h	Spacing of horizontal shear reinforcement to resist in-plane shear
S_v	Spacing of vertical shear reinforcement to resist in-plane shear
T	Torsion (kNm)
v_i	Shear stress in concrete wall section due to V_i
v_{ci}	Design concrete shear stress in wall section for in-plane bending
v_{co}	Design concrete shear stress in wall section for out-of-plane bending
v_{oh}	Shear stress in concrete wall section due to V_{OH}
v'_{ci}	Modified design concrete shear stress for in-plane bending
v'_{co}	Modified design concrete shear stress for out-of-plane bending
V_i	Combined in-plane flexural shear and torsional shear
V_I	In-plane shear force in a wall section
V_{si}	Shear resistance of shear reinforcement for in-plane shear
V_{so}	Shear resistance of shear reinforcement for out-of-plane shear
V'_{ci}	Available concrete shear strength for in-plane bending after allowing for V_{OH}
V'_{co}	Available concrete shear strength for out-of-plane bending after allowing for V_i
V_{OH}	Out-of-plane shear about horizontal plane
V_{OV}	Out-of-plane shear about vertical plane
x	Depth of neutral axis from compression face
X_i	Shear flow in the components of a closed cell (kN/m)
z	Depth of lever arm

β	Coefficient to determine effective height of wall
β	Factor for determination of deflection due to slenderness of wall
θ	Rate of twist (radians per metre length of member)
ψ	Factor to determine effective width of flange of shear wall

8.1 ANALYSIS OF WALLS

8.1.1 Walls and properties of walls

8.1.1.1 Definitions



SK 8/1 Plan and elevation of concrete wall.

Wall is a vertical load-bearing member whose length exceeds four times its thickness.

Unbraced wall is designed to carry lateral loads (horizontal loads) in addition to vertical loads.

Braced wall does not carry any lateral loads (horizontal loads). All horizontal loads are carried by principal structural bracings or lateral supports.

Reinforced wall contains at least the minimum quantities of reinforcement.

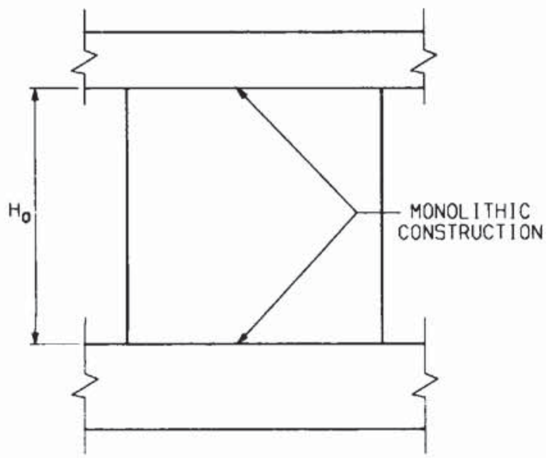
Plain wall contains either no reinforcement or less than the minimum quantity of reinforcement.

Stocky wall is where the effective height (H_e) divided by the thickness (h) does not exceed 15 for a braced wall and 10 for an unbraced wall.

Slender wall is a wall other than a stocky wall.

8.1.1.2 Effective heights

8.1.1.2.1 Reinforced wall – monolithic construction



SK 8/2 Wall monolithically constructed with slab and foundation.

$$H_e = \beta H_o$$

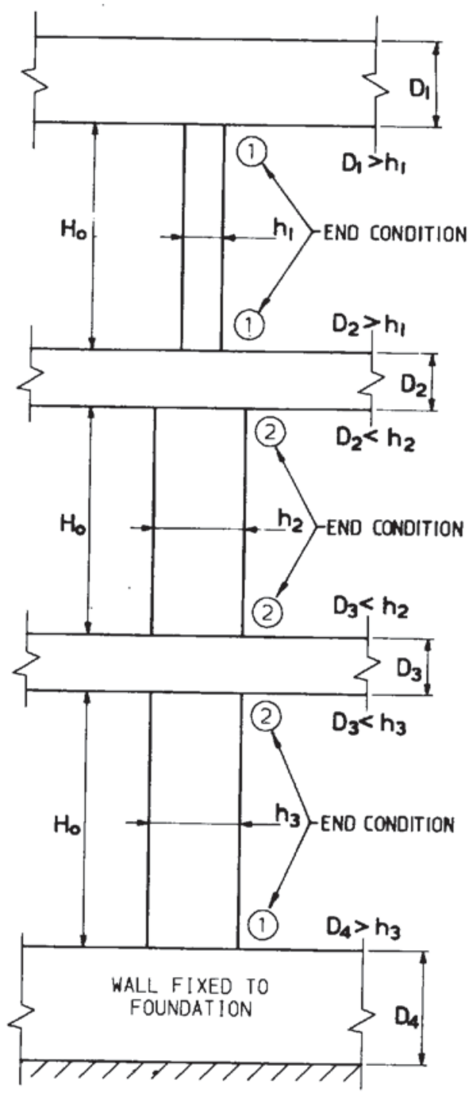
where H_o = clear height of wall.

Values of β for braced walls.

End condition at top	End condition at bottom	
	1	2
1	0.75	0.80
2	0.80	0.85

Values of β for unbraced walls.

End condition at top	End condition at bottom	
	1	2
1	1.2	1.3
2	1.3	1.5



SK 8/3 Wall/slab construction showing end conditions.

WALLS MONOLITHIC WITH SLAB OR FOUNDATION

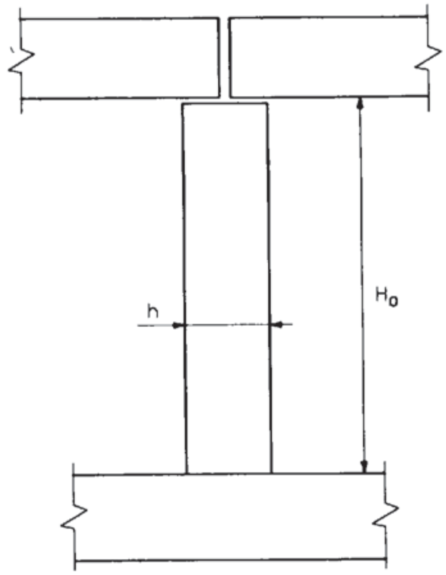
8.1.1.2.2 Reinforced wall – simply supported construction

$H_e = 0.75H_o$ for braced wall where lateral support resists lateral movement and rotation

$H_e = H_o$ for braced walls where lateral supports resist lateral movement

$H_e = 1.5H_o$ for unbraced wall with a roof slab or a floor slab at top

$H_e = 2.0H_o$ for unbraced wall with other forms of construction at top



SK 8/4 Slab simply supported on wall.

8.1.1.2.3 Reinforced wall – cantilever construction

$H_e = 2.0H_o$ for moment connection at foundation

8.1.1.2.4 Braced plain wall

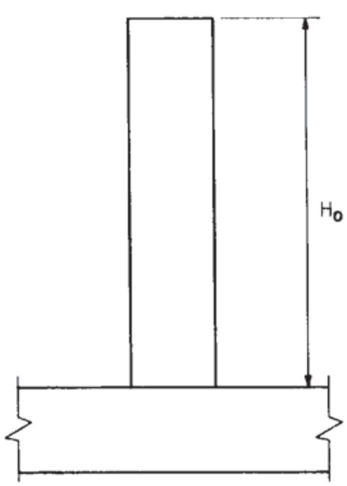
With translation and rotation restraint at any lateral support:

$H_e = 0.75H_o$

With translation restraint only at any lateral support:

$H_e = H_o$

Cantilever construction:



SK 8/5 Cantilever wall.

$H_e = 2H_o$ for rotational and lateral restraint at foundation

8.1.1.2.5 Unbraced plain wall

Supporting a roof or a floor slab:

$H_e = 1.5H_o$

For other walls with lateral restraints:

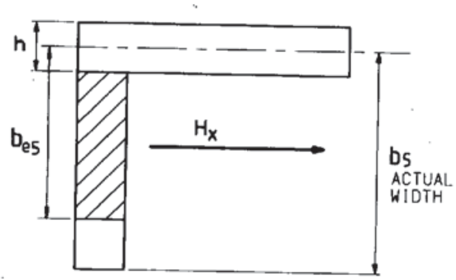
$H_e = 2.0H_o$

For cantilever plain wall:

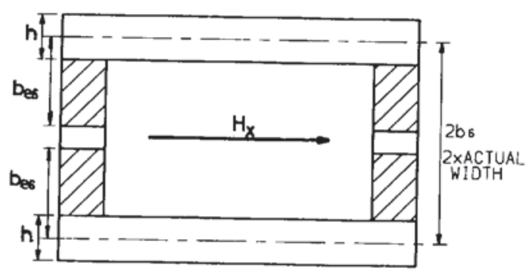
$H_e = 3.0H_o$

8.1.1.3 Effective width of flanges for in-plane bending

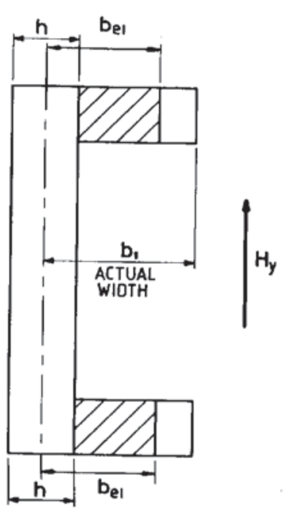
The effective width is width of wall perpendicular to direction of horizontal loading which is considered as effective as compression flange, and also vertical reinforcement provided in this width acts in tension as in tension flange. These factors for effective width are based on the recommendations in BS 5400: Part 5.[3]



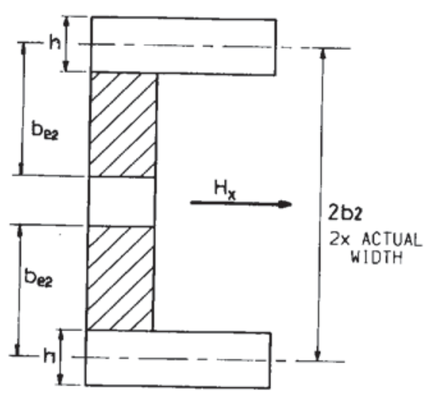
SK 8/6 Effective width of flange on plan of wall arrangement.



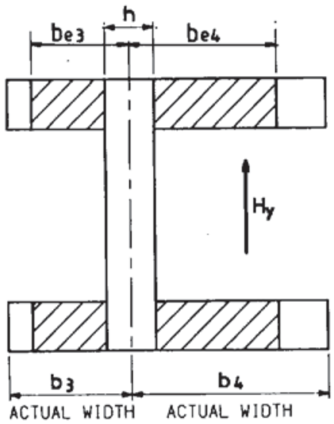
SK 8/7 Effective width of flanges of a closed cell on plan.



SK 8/8 Effective width of flanges of a channel shaped shear wall on plan.



SK 8/9 Effective width of flanges of a channel shaped shear wall on plan.



SK 8/10 Effective width of flanges of an I-shaped shear wall on plan.

From diagrams of typical shear wall sections:

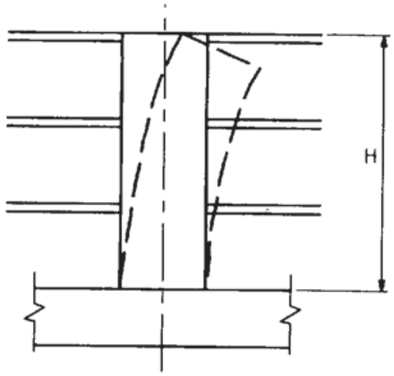
$b_{e1} = 0.85\ \psi b_1$
$b_{e2} = \psi b_2$
$b_{e3} = 0.85\ \psi b_3$
$b_{e4} = 0.85\ \psi b_4$
$b_{e5} = 0.85\ \psi b_5$
$b_{e6} = \psi b_6$

Effective breadth ratio ψ for shear walls (see BS 5400: Part 5[3]).

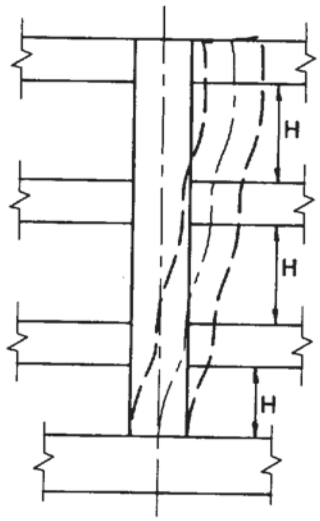
b/H	Uniformly distributed loading		Point loading at top	
	Cantilever wall	Continuous wall	Cantilever wall	Continuous wall
0	1.0	1.0	1.0	1.0
0.05	0.82	0.77	0.91	0.84
0.10	0.68	0.58	0.80	0.67
0.20	0.52	0.41	0.67	0.49
0.40	0.35	0.24	0.49	0.30
0.60	0.27	0.15	0.38	0.19
0.80	0.21	0.12	0.30	0.14
1.00	0.18	0.11	0.24	0.12

b = actual width of flange
H = height of cantilever walls, or
= half height between monolithic horizontal restraints for continuous walls

Note: The flange width limitations by use of a factor ψ are required to take into account shear lag effects. For ultimate limit state analysis, effects of shear lag in compression flange are sometimes ignored, but effective tension reinforcement in flange for in-plane bending should be limited within effective flange width as given by above expressions.



SK 8/11 Cantilever shear wall.

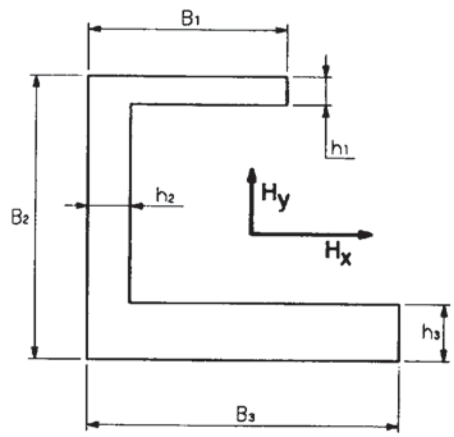


SK 8/12 Continuous shear wall.

8.1.1.4 Moment of inertia and shear area

The moment of inertia and shear area to be used for the computation of deflections of a cantilever shear wall structure and also for input to a computer program with a view to finding the interaction with other walls and frame structures could follow the typical suggestions given below.

Type 1 shear wall



SK 8/13 Type 1 shear wall.

for horizontal force H_x,

$$I_y = \frac{1}{12}h_1B_1^3 + \frac{1}{12}h_3B_3^3$$

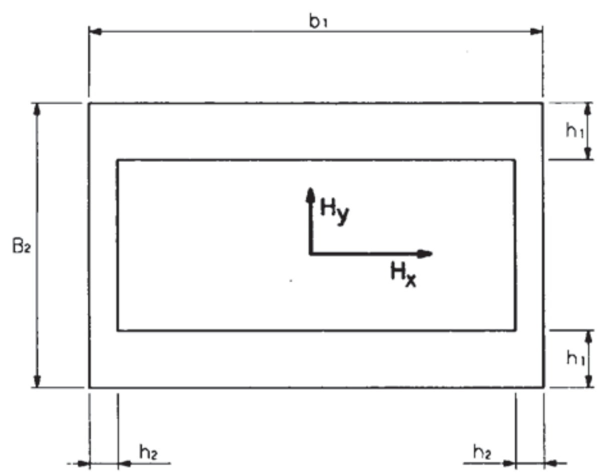
Shear area $= 0.8\ (B_1h_1 + B_3h_3)$

For horizontal force H_y,

$$I_x = \frac{1}{12}h_2B_2^3$$

Shear area $= 0.8B_2h_2$

Type 2 shear wall



SK 8/14 Type 2 shear wall.

For horizontal force H_x,

$$I_y = \frac{1}{12}\ (2h_1B_1^3)$$

Shear area $= 0.8\ (2h_1B_1)$

For horizontal force H_y,

$$I_x = \frac{1}{12}\ (2h_2B_2^3)$$

Shear area $= 0.8\ (2h_2B_2)$

The above philosophy may be applied to any shape and size of shear wall layout in a building. The stiffness of walls lying parallel to the direction of loading may only be included in the computation.

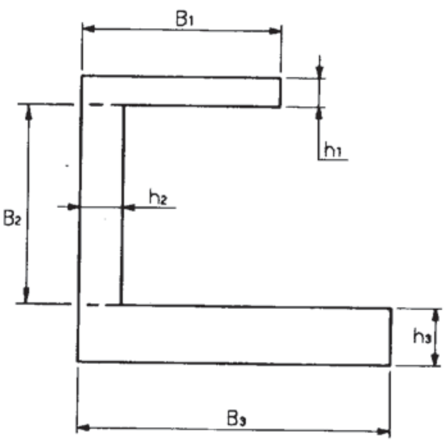
Note: The flanges of the shear walls have been ignored, as in T-beams in building frames, because the horizontal loads are generally of a reversible nature

and concrete in alternate flanges goes into tension. Considering cracked section moment of inertia including effective width of compression flanges does not produce too dissimilar results.

The out-of-plane stiffness of walls may be ignored in the global 3-D frame analysis.

8.1.1.5 Torsional stiffness

8.1.1.5.1 Open cell shear wall



SK 8/15 Open cell shear wall.

The torsional stiffness of individual wall elements should be added. The torsional stiffness of the open cell as a whole is

$$C = c_1 h_1^3 B_1 + c_2 h_2^3 B_2 + c_3 h_3^3 B_3$$

Values of coefficient c.

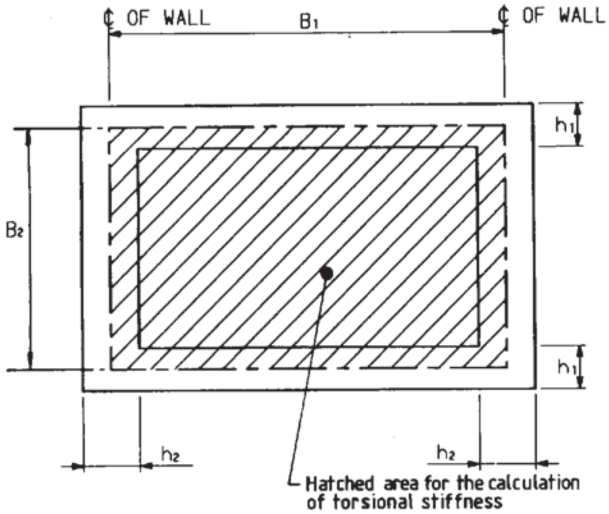
B/h	1	1.5	2	3	5	10
c	0.14	0.20	0.23	0.26	0.29	0.31

Note: In a global 3-D model each wall of the open cell shear wall may be modelled separately as vertical stiffness elements. The property of each wall will then include the individual torsional stiffness expressed as $C = ch^3B$.

8.1.1.5.2 Single closed cell shear wall

Torsional stiffness,

$$J = \frac{4A^2}{\Sigma(B/h)} = \frac{4A^2}{\dfrac{2B_1}{h_1} + \dfrac{2B_2}{h_2}}$$

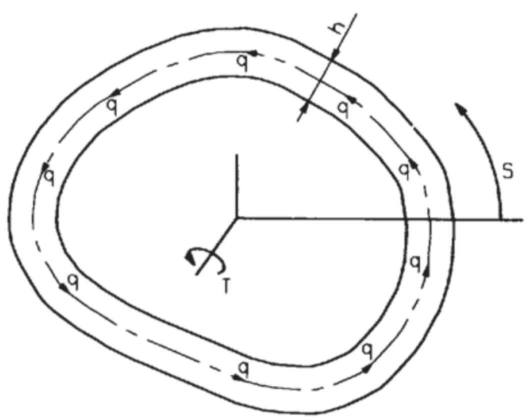


SK 8/16 Single closed cell shear wall.

$$A = B_1 B_2 \quad \text{(area bounded by median line)}$$

$$T = 2Aq$$

where T = torsion applied
q = shear flow (kN/m).



SK 8/17 Closed irregular cell section subject to torsion.

The general formula for any single closed cell is given by:

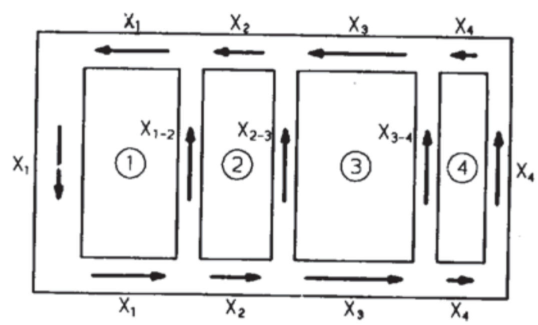
$$J = \frac{4A^2}{\int \frac{ds}{h}}$$

where A = area bounded by median line of wall thickness
h = thickness of wall
s = median length of wall.

$$J = \frac{T}{G\theta}$$

where T = torque applied = $2Aq$
G = modulus of rigidity = $E/2(1 + \mu)$
E = modulus of elasticity
θ = rate of twist in radians per metre length = $(q/2AG) \int(\mathrm{d}s/t)$
q = shear flow (kN/m).

8.1.1.5.3 Multi-cell closed shear wall



SK 8/18 Closed multiple-cell section subject to torsion.

General equation for unit twist of one cell:

$$\theta_i = \frac{1}{2A_iG}\left(X_i \int_i \frac{\mathrm{d}s}{t} - X_{i-1} \int_{i-1,i} \frac{\mathrm{d}s}{t} - X_{i+1} \int_{i,i+1} \frac{\mathrm{d}s}{t}\right)$$

$$p_i = \int_i \frac{\mathrm{d}s}{t} \qquad p_{i-1,i} = \int_{i-1,i} \frac{\mathrm{d}s}{t} \qquad p_{i,i+1} = \int_{i,i+1} \frac{\mathrm{d}s}{t}$$

For compatibility, assume $\theta_i = \theta$

$$\therefore \quad -X_{i-1}p_{i-1,i} + X_ip_i - X_{i+1}p_{i,i+1} = 2A_iG\theta$$

Assume $X_i' = X_i/2G\theta$

$$\therefore \quad -X_{i-1}'p_{i-1,i} + X_i'p_i - X_{i+1}'p_{i,i+1} = A_i$$

From this general equation:

$$\begin{array}{llll}
X_1'p_1 & - X_2'p_{1,2} & & = A_1 \\
- X_1'p_{1,2} & + X_2'p_2 & - X_3'p_{2,3} & = A_2 \\
\cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\
- X_{n-2}'p_{n-2,n-1} & + X_{n-1}'p_{n-1} & - X_n'p_{n-1,n} & = A_{n-1} \\
 & - X_{n-1}'p_{n-1,n} & + X_n'p_n & = A_n
\end{array}$$

Solving for the unknowns in the above matrix gives values of X_1' to X_n'.

$$T = 4G\theta\sum_1^n A_iX_i'$$

When T is known, θ can be found.

$$J = \frac{T}{G\theta} = 4\sum_{1}^{n} A_i X_i'$$

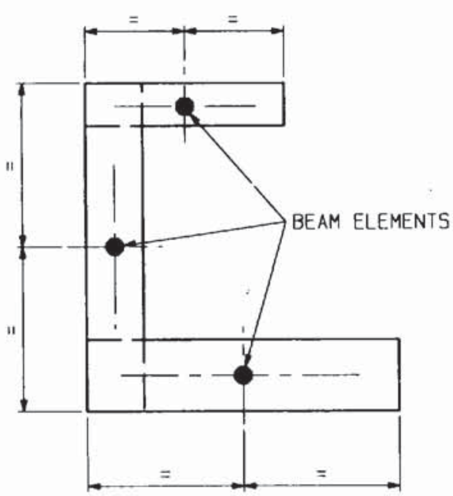
$$\text{Shear flow, } X_i = 2G\theta X_i' = \left(\frac{2T}{J}\right) X_i'$$

$$X_{i,\,i+1} = X_i - X_{i+1}$$

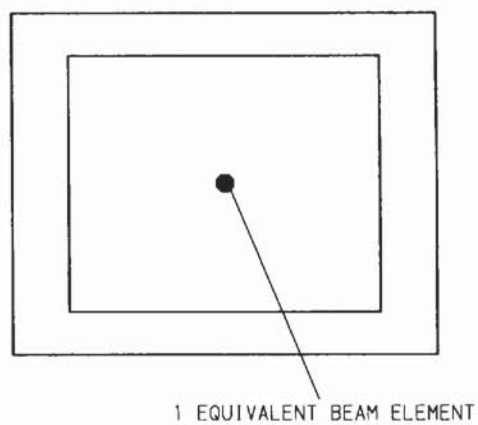
8.1.2 Modelling for structural analysis

8.1.2.1 Global analysis for in-plane forces

Modelling as individual walls
Each individual wall can be modelled as a vertical beam element with properties as described in Section 8.1.1.4 and 8.1.1.5.



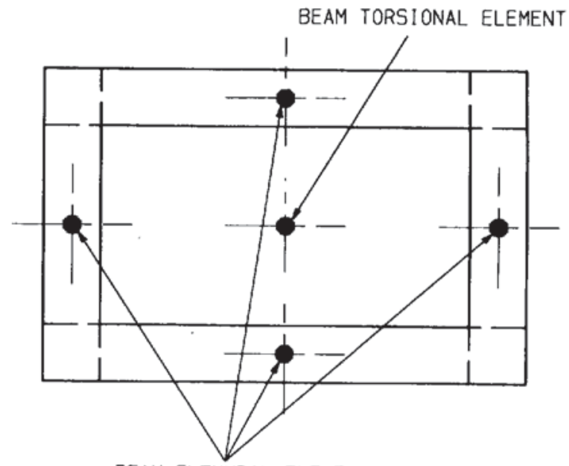
SK 8/19 Walls of a shear wall system converted to equivalent beam elements.



SK 8/20 A closed cell converted to one equivalent beam element.

Modelling as a combined unit
A set of walls can be combined to be represented by one beam element. In this case the property of this beam element will be the summation of properties for individual walls. The representative beam element may be located at the CG of the wall configuration.

In the case of closed cell shear wall structure the equivalent torsional stiffness will not be the sum of the individual torsional stiffnesses of the walls. The equivalent torsional stiffness will be found as per Section 8.1.1.5.2. When a closed cell shear wall structure is modelled as individual wall elements, then the torsional stiffness parameters for these individual wall elements will be considered as negligible. A separate single beam



SK 8/21 Cell converted to a combination of flexural and torsional elements.

element must be modelled to represent the torsional stiffness of the closed cell system. This beam element will have no bending or shear stiffness but only torsional stiffness. This element may be placed at the CG of the closed cell structure. This separate torsional beam element will be connected by rigid offsets with the individual wall bending elements.

The design of walls should be carried out on an individual wall basis. The determination of individual wall moments and shears from the representative single beam element will be carried out by using the relative bending and shear stiffnesses of individual walls.

8.1.2.2 Local analysis for out-of-plane forces

Out-of-plane forces on the wall may be due to the following:

- Eccentric dead and live load.
- Wind pressure on wall panel.
- Earthquake wall mass excitation.
- Earth pressure on wall face.
- Water pressure on wall face.
- Thermal gradient across wall thickness.

Local analyses should be carried out using appropriate boundary conditions. Published tables may be used to find out-of-plane bending moments and shears. Combined bending moment triads using the Wood–Armer principle should be used to find the reinforcement requirement.

For out-of-plane local analyses, follow the general guidelines in Section 3.1.

For a complicated wall geometry, wall panels in the out-of-plane direction may be modelled using hypothetical grillage elements using for solution a grillage suite of a computer software.

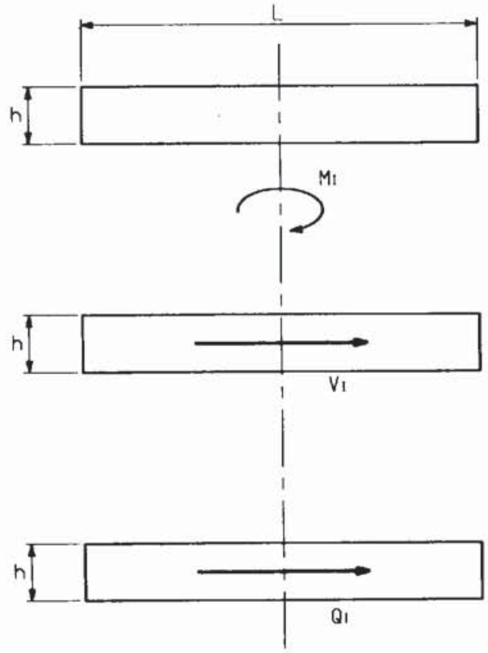
8.2 STEP-BY-STEP DESIGN PROCEDURE FOR WALLS

Step 1 ***Find properties of wall system***
Find moment of inertia and shear area (follow Section 8.1.1.4).

Step 2 ***Find torsional stiffness of wall system***
Follow Section 8.1.1.5.

Step 3 ***Carry out modelling for analysis***
Follow Sections 8.1.2.1 and 8.1.2.2.

Step 4 ***Carry out global analysis***

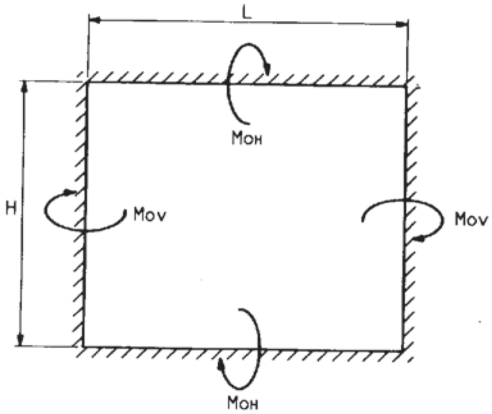


SK 8/22 Description of in-plane forces in a wall.

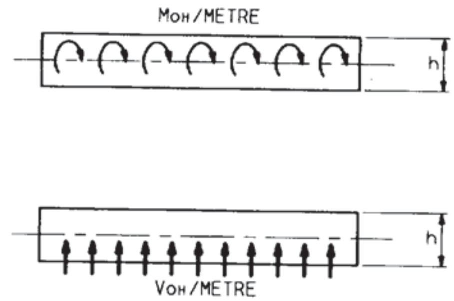
Find in-plane forces in walls. After analysis the following internal forces should be available for each individual wall section in the system.

M_I = in-plane bending moments
V_I = in-plane shear force
Q_I = in-plane shear flow due to torsion
N = axial load

Step 5 ***Carry out local analysis***
Find out-of-plane forces in walls (follow Section 8.1.2.2).
After analysis the following internal forces should be available for each individual wall panel in the system.



SK 8/23 Elevation of a wall panel showing out-of-plane moments.



SK 8/24 Out-of-plane internal forces in a wall section.

M_{OH} = out-of-plane bending moment about horizontal plane
M_{OV} = out-of-plane bending moment about vertical plane
V_{OH} = out-of-plane shear about horizontal plane
V_{OV} = out-of-plane shear about vertical plane

Step 6 ***Carry out combination of loading***

This should preferably be carried out in a tabular fashion for different load cases. The load combinations should be generally as follows:

$$LC_1 = 1.4DL + 1.6LL + 1.4EP + 1.4WP$$
$$LC_2 = 1.0DL + 1.4EP + 1.4WP$$
$$LC_3 = 1.4DL + 1.4WL + 1.4EP + 1.4WP$$
$$LC_4 = 1.0DL + 1.4WL + 1.4EP + 1.4WP$$
$$LC_5 = 1.2DL + 1.2LL + 1.2WL + 1.2EP + 1.2WP$$

Note: Load combinations LC_2 and LC_4 should be considered only when dead and live load have beneficial effects.

where DL = dead load
LL = live or superimposed load
WL = wind load or earthquake load
WP = water pressure
EP = earth pressure.

Step 7 ***Check slenderness of wall***

Determine type of wall: braced, unbraced, plain or reinforced.
Find effective height (follow Section 8.1.1.2.1)

$$H_e = \beta H_o$$

Check slenderness ratio H_e/h.
For braced reinforced wall with $< 1\%$ reinforcement, limit of $H_e/h \leq 40$.

For braced reinforced wall with $\geq 1\%$ reinforcement, limit of $H_e/h \leq 45$.
For unbraced reinforced wall and plain wall, limit of $H_e/h \leq 30$.

If $H_e/h \leq 15$ (braced) or 10 (unbraced), then design as a *stocky wall*. Otherwise, design as *slender wall*.

Step 8 ***Find effective width of flanges for reinforced wall***
Follow Section 8.1.1.3.

Step 9 ***Find additional out-of-plane moments about horizontal plane***

(1) Moments due to minimum eccentricity of $h/20$ or 20 mm of direct loads from beams and slabs simply supported on wall.
(2) Moments due to slenderness of wall.

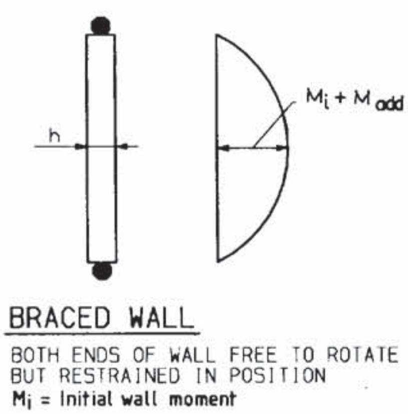
For $H_e/h > 15$ (braced) or >10 (unbraced):

Note: Wall braced or unbraced in the transverse direction only to be considered for additional moments.

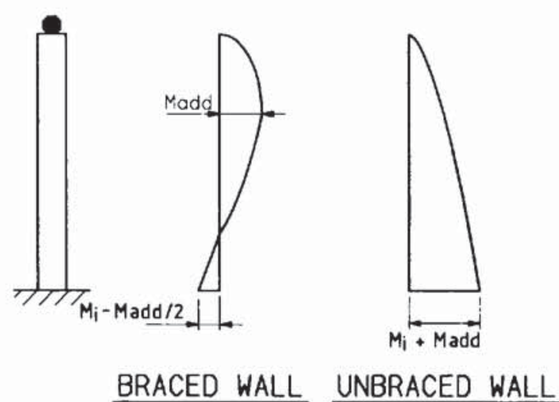
Deflection due to slenderness of wall, $a_u = \beta\ K\ h$

Assume $K = 1$ for conservatism.

$$\beta = \frac{1}{2000}\left(\frac{H_e}{h}\right)^2$$



SK 8/25 Wall additional moment.



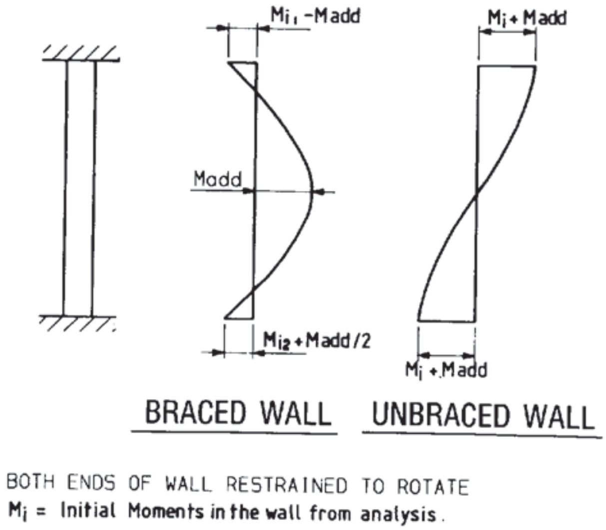
SK 8/26 Wall additional moments.

Additional moment due to slenderness, $M_{add} = Na_u$

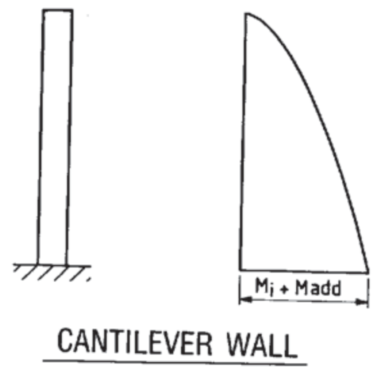
where N = direct ultimate load on wall.

Combine this additional moment, M_{add}, with any other out-of-plane moments obtained from analysis using Figure 3.20 or Figure 3.21 of BS 8110: Part 1: 1985.[1]

Note: These additional moments should be doubled if the wall has only one central layer of reinforcement.



SK 8/27 Wall additional moments.



SK 8/28 Wall additional moments.

These out-of-plane bending moments and shears are about a horizontal plane.

Step 10 ***Design stocky braced reinforced wall with approximately symmetrical arrangement of slabs***

Spans of slab on either side of wall within ± 15% and slab subjected to uniform load.

$n_w \le 0.35 f_{cu} A_c + 0.67 A_{sc} f_y$

where n_w = total design ultimate axial load on wall.

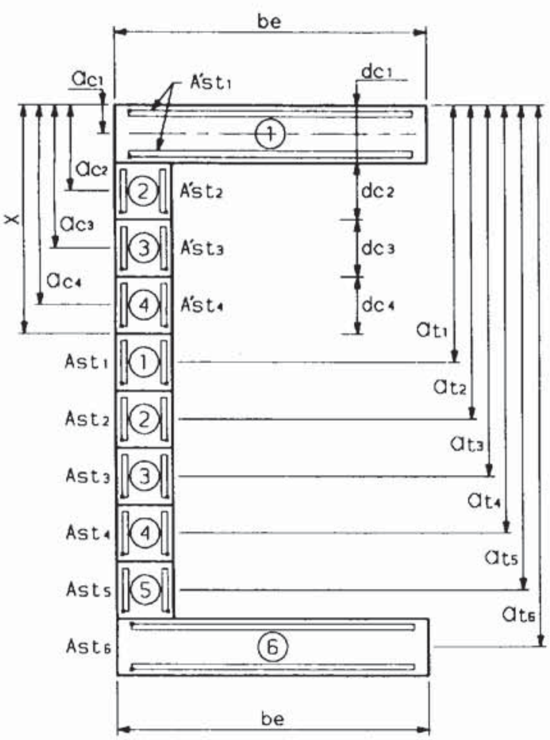
Step 11 ***Determine cover to reinforcement***

Determine cover to reinforcement as per Tables 11.6 and 11.7.

Step 12 ***Design of reinforced wall – rigorous method***

Using the effective flange widths found in Step 8, find by elastic analysis the stresses in the concrete and steel due to in-plane bending moment and axial load only.

(1) Assume initially 0.40% area of steel in wall distributed uniformly in two layers on two faces.
(2) Assume a value of x for depth of neutral axis from compression face.
(3) Divide compression zone into layers of concrete with depths d_{c1}, d_{c2}, d_{c3}, etc. and find centres of these layers from compression face a_{c1}, a_{c2}, a_{c3}, etc.
(4) Conveniently group bars in tension zone and find area of groups A_{st1},



SK 8/29 A typical example of analysis of a shear wall.

A_{st2}, A_{st3}, etc. and also distances of these groups of bars from compression face, i.e. a_{t1}, a_{t2}, a_{t3}, etc.

(5) Find the following:

$$A_T = \frac{\Sigma A_{st} a_t (a_t - x)}{\Sigma A_{st}(a_t - x)}$$

$$S_1 = A_{c1} + (m - 1)A'_{st1}$$

where A_{c1} = area of concrete in the layer 1 of concrete in compression zone

A'_{st1} = area of compressive reinforcement in layer 1 of concrete in compression zone

m = modular ratio = E_s/E_c.

$$A_c = \frac{\Sigma(x - a_c)a_c S}{\Sigma(x - a_c)S}$$

$$\bar{x} = \frac{m\Sigma A_{st} a_t + \Sigma S a_c}{m\Sigma A_{st} + \Sigma S}$$

$$e = \frac{M}{N} = \frac{\text{in-plane bending moment}}{\text{axial compression}}$$

$$f_c = \frac{Nx(e + A_T - \bar{x})}{(A_T - A_c)\,\Sigma\,(x - a_c)S}$$

$$f_{st} = \frac{(d - x)}{\Sigma(a_t - x)A_{st}}\left[\frac{f_c}{x}\,\Sigma(x - a_c)S - N\right]$$

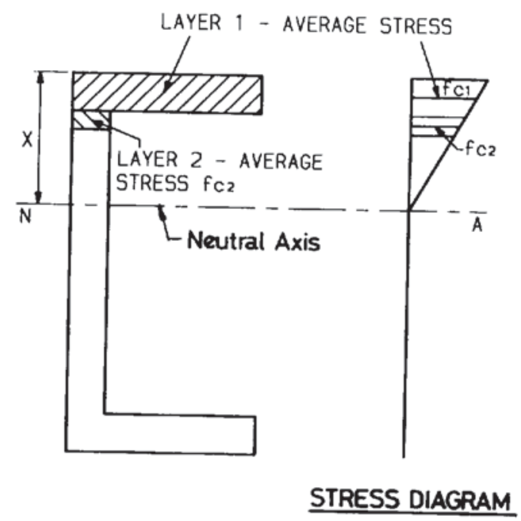
(6) Finally check:

$$x = \frac{d}{1 + \frac{f_{st}}{mf_c}}$$

If x is different from the assumed value then repeat the exercise with a new assumed x until convergence is reached.

(7) Find final stresses f_c and f_{st} after convergence. If f_{st} is greater than $0.87f_y$, then increase area of steel by proportion $f_{st}/0.87f_y$. If f_c is greater than $0.45f_{cu}$, then increase thickness of wall.

(8) Find revised f_c and f_{st} with increased reinforcement. There will be no need to carry out the iteration to find x with increased reinforcement.



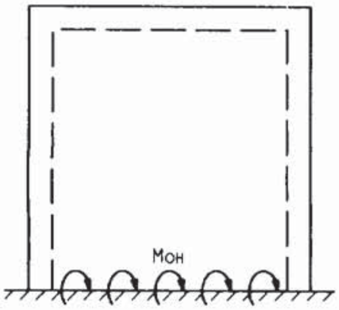
SK 8/30 Elastic stress analysis of a shear wall.

(9) Draw stress diagram for in-plane bending moment and direct axial load. Divide wall into unit lengths. Over each unit length convert the average compressive stress in compression zone into a direct load by multiplying with the area of the unit length of wall. This compression force acts in combination with the out-of-plane bending moment in that length of wall. Design reinforcement for out-of-plane bending additional to that already provided in that unit length using Tables 11.8 to 11.17 – design tables for rectangular columns.

(10) In the tension zone of the wall subject to in-plane bending moment and axial load only, assume the concrete as unstressed. Find reinforcement required for out-of-plane bending moment as in an RC beam following Step 10 of Section 2.3. Add this reinforcement to reinforcement already provided for in-plane bending moment.

If reinforcement provided for in-plane bending is not fully stressed to the ultimate limit of $0.87f_y$, then the residual capacity of this reinforcement may be used to withstand out-of-plane bending moment.

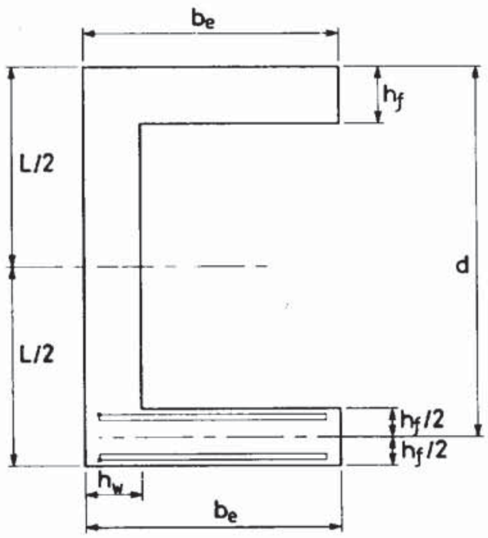
Average tensile strain in the tensile flange may be found and converted to an average tensile force in the flange for computation of reduced shear stress for out-of-plane bending. Conservatively ignore concrete shear resistance in tension flange.



SK 8/31 Out-of-plane bending of a panel of a wall.

Note: Each panel of wall should be checked for global loads in both orthogonal directions separately if these loads are not acting simultaneously. The worst reinforcement from either of the two orthogonal loads will be used. The out-of-plane bending moments for combination with in-plane bending moments are about the horizontal plane.

Step 13 Design of reinforced wall – simple method



SK 8/32 Analysis of a shear wall against in-plane bending.

Flanged wall

$$M' = M + N\left(\frac{L}{2} - \frac{h_f}{2}\right)$$

where M = in-plane bending moment
N = axial load.

$$K = \frac{M'}{f_{cu}bd^2} \leq 0.156$$

$$z = d\left[0.5 + \sqrt{\left(0.25 - \frac{K}{0.9}\right)}\right] \le 0.95d$$

$$x = \frac{d - z}{0.45}$$

$$A_s = \left(\frac{M'}{0.87f_y z}\right) - \left(\frac{N}{0.87f_y}\right)$$

This reinforcement will be provided in effective width of flange in two layers as shown. The web of the flanged wall will have the minimum reinforcement unless dictated by out-of-plane bending moments or reinforcement requirement as part of tension flange for other direction of orthogonal load.

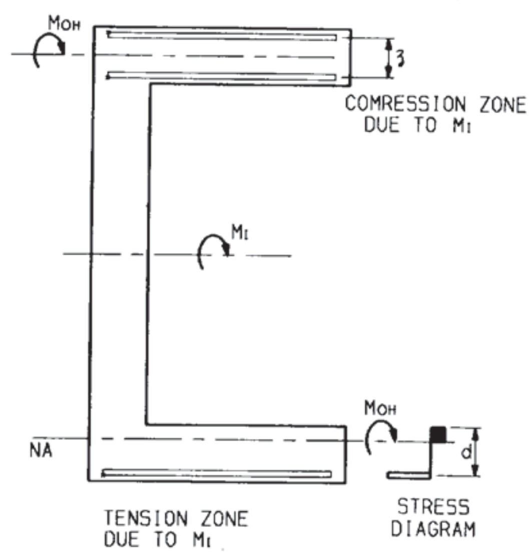
If $x > h_f$, then follow Step 11 of Section 2.3.

The out-of-plane bending about a horizontal plane on either the wall flange or the wall web may be due to the following:

(1) Out-of-plane framing action with supported slab.
(2) Slenderness of wall.
(3) Eccentric loads from beam or slab or any other structure on wall.
(4) Coacting horizontal loads on wall panel due to wind, earthquake, water pressure or earth pressure.
(5) Thermal gradient across wall thickness.

The reinforcement for the out-of-plane bending moment about a horizontal plane will be calculated as follows.

In the compression zone of concrete wall due to in-plane bending moment, assume that the concrete has already reached the ultimate stage



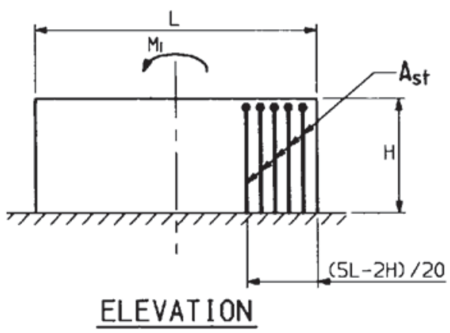
SK 8/33 Analysis of a shear wall against in-plane and out-of-plane bending.

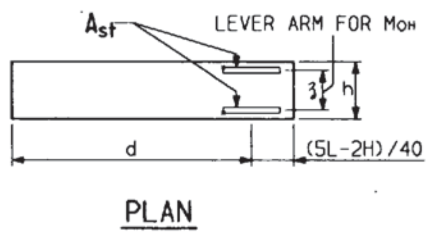
and cannot take any more load. Hence, the bending moment will be resisted by equal amounts of compressive and tensile steel with a lever arm equal to the distance between the two layers of steel.

In the tension zone of concrete wall due to in-plane bending moment, assume that the concrete is unstressed and use the beam theory to find reinforcement due to out-of-plane bending moment.

The reinforcement required due to out-of-plane bending moment will be added to the reinforcement found for in-plane bending.

Step 14 Design of reinforced wall – short and squat cantilever – deep-beam approach





SK 8/34 Design of a shear wall by deep-beam method.

This approach may be used for walls with total height less than or equal to their length. For in-plane bending consider the wall as a deep-beam and follow the deep-beam theory of stress distribution.

For horizontal loading to resist in-plane bending moment

$$z = \frac{6H}{5} \quad \text{when } \frac{H}{L} \leq 0.5$$

$$= \frac{2(H+L)}{5} \quad \text{when } 0.5 < \frac{H}{L} \leq 1$$

Tension reinforcement to be distributed over a length of wall equal to

$$\frac{5L - 2H}{20}$$

$$d = L - \left(\frac{5L - 2H}{40}\right)$$

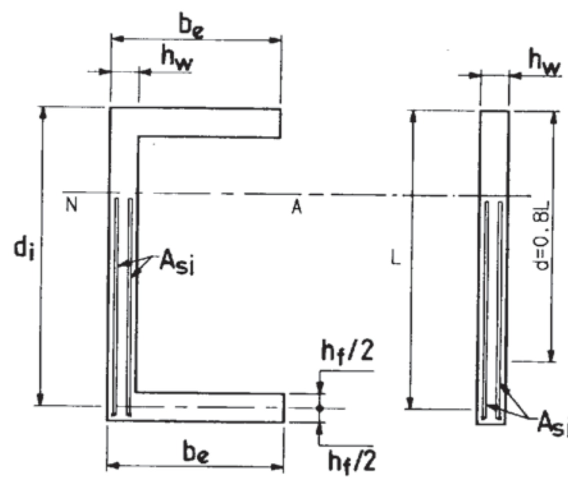
$$M' = M + N\left(\frac{L}{2} - \frac{5L - 2H}{40}\right)$$

$$A_s = \left(\frac{M'}{0.87 f_y z}\right) - \left(\frac{N}{0.87 f_y}\right)$$

Note: The flexural strain in concrete is very small in a short and squat cantilever wall and for all practical purposes may be ignored when designing for the

transverse out-of-plane bending moment. Use the normal beam theory to find reinforcement for the transverse out-of-plane bending moment.
Add this additional reinforcement for transverse out-of-plane bending moment to reinforcement already found for in-plane bending moment. The out-of-plane bending moment in this context is about the horizontal plane.
The flanges of the wall, if present, either in tension or compression may be ignored if this deep-beam approach is used. The shear strains in a wall with the aspect ratios of a deep beam may be high and a conservative approach taking the shear-lag effect would be to ignore the flanges.

Step 15 Check shear



SK 8/35 Design for in-plane shear.

In-plane bending

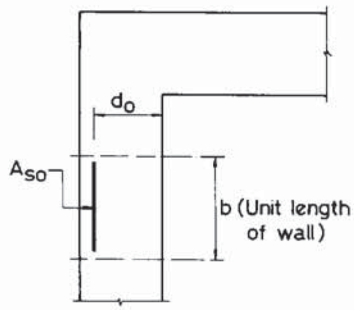
$$p_i = \frac{100A_{si}}{h_w d_i}$$

where p_i = percentage of tensile reinforcement in in-plane direction
A_{si} = reinforcement available to resist in-plane bending
d_i = effective depth as shown.

$$v_i = \frac{V_i}{h_w d_i} \leq 0.8\sqrt{f_{cu}} \leq 5\,\text{N/mm}^2$$

where v_i = shear stress due to in-plane bending and torsional shear flow in wall
V_i = combined in-plane shear
= flexural shear + torsional shear
= $V_1 + Q_1$ (see Step 4).

v_{ci} = design concrete in-plane shear stress depending on p_i and f_{cu} (see Figs 11.2 to 11.5)



SK 8/36 Design for out-of-plane shear.

Out-of-plane bending

$$p_o = \frac{100A_{so}}{bd_o}$$

where A_{so} = reinforcement available to resist out-of-plane bending
b = unit length horizontally
d_o = effective depth in transverse out-of-plane direction
p_o = percentage of tensile reinforcement in out-of-plane direction.

v_{co} = design concrete out-of-plane shear stress depending on p_o and f_{cu} (see Figs 11.2 to 11.5)

V_{OH} = out-of-plane shear about a horizontal plane over a unit width b.

$$v_{oh} = \frac{V_{OH}}{bd_o} \leq 0.8\sqrt{f_{cu}} \leq 5\,\text{N/mm}^2$$

No shear reinforcement is necessary if the following equation is satisfied:

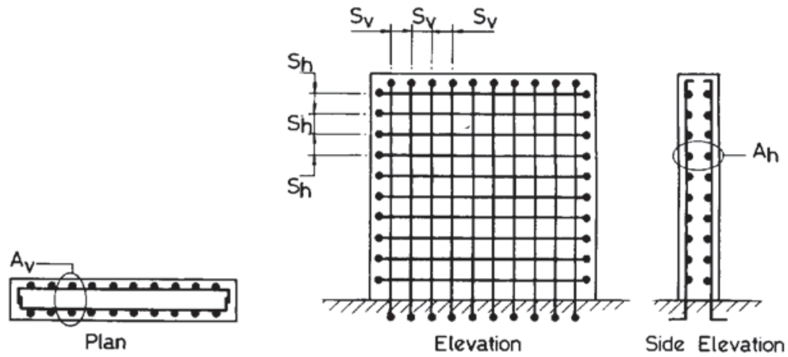
$$\frac{v_i}{v_{ci}} + \frac{v_{oh}}{v_{co}} \leq 1$$

Note: In Step 12 and Step 13 the wall is designed as flanged beams for in-plane loading. For out-of-plane shear in the flanges which acts together with the in-plane loading, the check should be carried out separately for compression and tension flange. For compression flange the enhancement of design shear stress due to axial load may be allowed based on average compressive stress. For tension flange the concrete may be conservatively ignored and the shear force will be totally carried by shear reinforcement. Alternatively, average tensile strain in concrete may be found and the shear stress reduction formula may be used.

Step 16 Calculate shear reinforcement

Note: Increase or decrease of design concrete shear stress due to presence of axial load may be allowed following the formula on page 160.

SK 8/37 In-plane shear reinforcement in walls.



Case 1: $v_{oh} < v_{co}$

$$v'_{ci} = \left(1 - \frac{v_{oh}}{v_{co}}\right) v_{ci}$$

where v'_{ci} = available concrete shear strength in in-plane direction for use with shear reinforcement.

$$V'_{ci} = v'_{ci} h_w d_i$$

Provide shear reinforcement in in-plane direction for a shear force equal to $(V_i - V'_{ci})$ and check:

$$(V_i - V'_{ci}) \leq V_{si}$$

$$V_{si} = \frac{0.87 A_h f_y d_i}{S_h}$$

where V_{si} = shear resistance of horizontal bars in wall for in-plane shear
S_h = spacing of horizontal bar in wall
A_h = area of horizontal shear reinforcement
f_y = characteristic yield strength of reinforcement.

Note: Provide equal amount of vertical shear reinforcement with horizontal shear reinforcement.

$$\frac{A_v}{S_v} = \frac{A_h}{S_h}$$

where A_v = area of vertical shear reinforcement
S_v = spacing of vertical shear reinforcement.

In this Case 1, no shear reinforcement is required for out-of-plane flexure. Provide the shear reinforcement for in-plane shear in addition to other bars required for in-plane and out-of-plane bending moments.

Case 2: $v_{oh} > v_{co}$

$$v'_{ci} = \frac{v_{ci}\, v_i}{v_{oh} + v_i}$$

$$v'_{co} = \frac{v_{co}\, v_{oh}}{v_{oh} + v_i}$$

where v'_{ci} = available concrete shear stress strength in in-plane direction for use with shear reinforcement

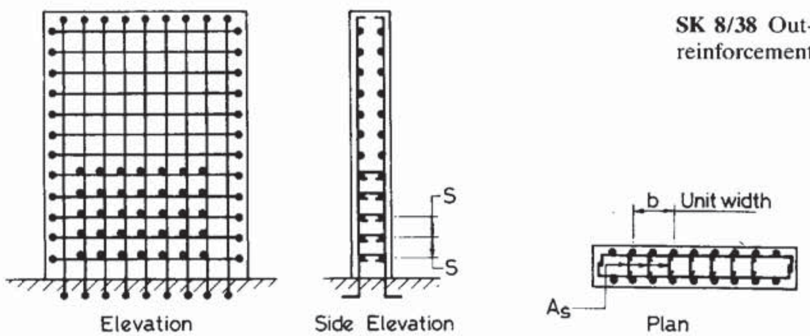
v'_{co} = available concrete shear stress strength in out-of-plane direction for use with shear reinforcement.

$$V'_{ci} = v'_{ci}\, hd_i$$

$$V'_{co} = v'_{co}\, bd_o$$

Provide shear reinforcement in in-plane direction, as in Case 1, which satisfies

$$V_{si} \geq (V_i - V'_{ci})$$



SK 8/38 Out-of-plane shear reinforcement in walls.

For shear reinforcement in out-of-plane horizontal direction, use links through thickness of wall.

For out-of-plane horizontal directional shear, resistance from links, V_{so}, for a unit length b is given by

$$V_{so} = \frac{0.87 f_y A_s d}{S}$$

where A_s = area of links over a unit width b

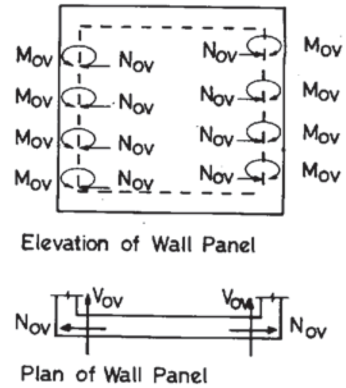
S = vertical spacing of links.

Check $V_{OH} - V'_{co} \leq V_{so}$

Note: If considerable ductility is required of a shear wall, as in seismic design, the whole shear force should be carried by reinforcement and the shear capacity of concrete may be ignored if the shear capacity of concrete is exceeded.

Step 17 Check-out-of-plane bending about vertical plane

After local analysis of wall panel the bending moments, direct loads and shears about the vertical plane in the panel are obtained.



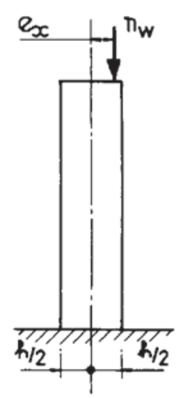
SK 8/39 Internal forces in wall panel due to out-of-plane loading.

Design horizontal reinforcement for flexure of wall panel about a vertical plane. The procedure is the same as in Step 4 of Section 3.3.

Check shear stress and reinforcement for shear as in Step 6 of Section 3.3.

Step 18 Design of plain (not adequately reinforced) walls

(A) Stocky braced plain wall



SK 8/40 Eccentric loading on wall in out-of-plane direction.

Check $n_w \leq 0.3\,(h - 2e_x)f_{cu}$

where n_w = maximum ultimate axial load per unit length on wall

e_x = resultant eccentricity of all loads at right angles to plane of wall (minimum value of e_x is $h/20$).

(B) Slender braced plain wall

Check $n_w \leq 0.3\,(h - 2e_x)f_{cu}$

and $n_w \leq 0.3\,(h - 1.2e_x - 2e_a)f_{cu}$

$$e_a = \frac{H_e^2}{2500h}$$

where H_e = effective height (as per Section 8.1.1.2).

(C) Unbraced plain wall

Check $n_w \leq 0.3\ (h - 2e_{x,1})f_{cu}$
and $n_w \leq 0.3\ [h - 2\ (e_{x,2} + e_a)]f_{cu}$

where $e_{x,1}$ = resultant eccentricity of loads at top of wall
$e_{x,2}$ = resultant eccentricity of loads at bottom of wall.

Step 19 Shear check of plain walls

Check $V \leq 0.25 n_w$

where V = in-plane ultimate shear force per unit length.

Check $v = \dfrac{V}{hb} \leq 0.45\,\text{N/mm}^2$

where v = shear stress
b = unit length (mm).

Note: A plain wall subjected to in-plane shear should satisfy at least one of the above checks.

Step 20 Check minimum reinforcement

Minimum compression (vertical) reinforcement in reinforced wall = 0.4% ($f_y = 460\,\text{N/mm}^2$) of gross cross section

Minimum horizontal tension reinforcement to withstand out-of-plane loads = 0.13% ($f_y = 460\,\text{N/mm}^2$) of gross cross-section on each face

Minimum anti-crack reinforcement = 0.25% ($f_y = 460\,\text{N/mm}^2$) of gross cross-section

Step 21 Check maximum reinforcement

Maximum vertical reinforcement in wall = 4% of gross cross-section

Step 22 Check containment of wall reinforcement

For vertical compression reinforcement in walls up to 2% of gross cross-sectional area, use the following minimum horizontal bars:

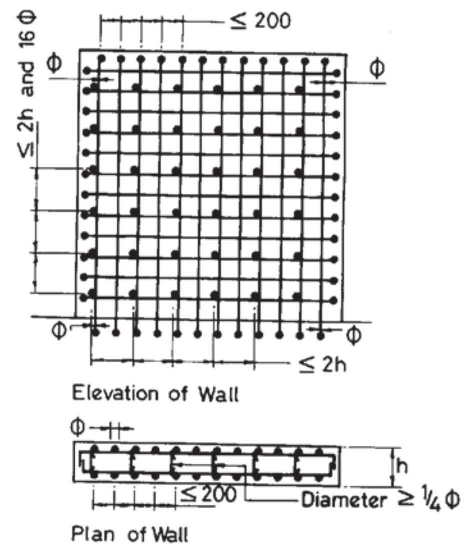
0.25% of gross concrete area

Horizontal bar diameter should be greater than or equal to $\frac{1}{4}$ size of vertical bars but not less than 6 mm diameter.
For vertical compression reinforcement in walls greater than 2% of gross cross-sectional area, use links through the thickness of wall.

Dia. of links $\geq \frac{1}{4}$dia. of vertical bars or 6 mm, whichever is greater

Spacing of links $\geq 2h$ in horizontal and vertical direction

Spacing of links in vertical direction should not be more than 16 times vertical bar diameter.



SK 8/41 Detailing rules for walls.

Any vertical compression bar not enclosed by link should be within 200 mm of a restrained bar.

Step 23 ***Check early thermal cracking***
See Step 14 of Section 3.3.

Step 24 ***Clear spacing of bars in tension***
Follow Step 13 of Section 3.3.

Step 25 ***Connections***
See Chapter 10.

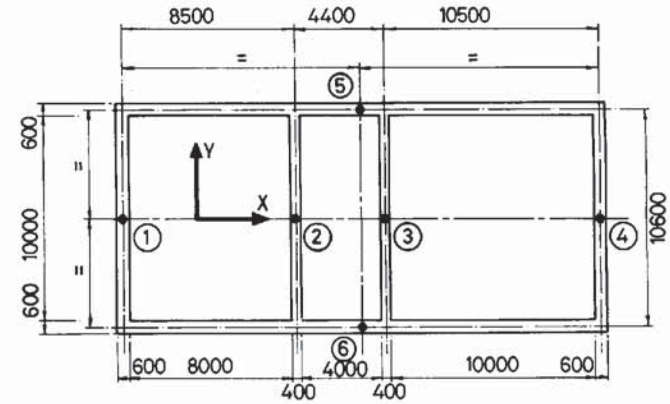
8.3 WORKED EXAMPLE

Example 8.1 ***Reinforced concrete cell***
Design the walls of a reinforced concrete cell which forms part of the horizontal stability system of a building.

Step 1 ***Find properties of wall system***
Follow Section 8.1.1.4.

Stiffness in y-direction
Divide the wall cell system into six beam elements located at the centroid of each wall.

Equivalent beam elements 1, 2, 3 and 4 contribute to stiffness in y-direction.



SK 8/42 Location of flexural vertical beam elements.

Equivalent beam elements 1 and 4 (ignoring flanges)

$$I_x = \frac{1}{12} \times 600 \times 11\,200^3 = 70.25\,\text{m}^4$$

Shear area $= 0.8 \times 600 \times 11\,200 = 5.38\,\text{m}^2$

Equivalent beam elements 2 and 3 (ignoring flanges)

$$I_x = \frac{1}{12} \times 400 \times 11\,200^3 = 46.84\,\text{m}^4$$

Shear area $= 0.8 \times 400 \times 11\,200 = 3.58\,\text{m}^2$

Note: The moments of inertia and shear area of equivalent beam elements 1, 2, 3, and 4 about the y-axis will be ignored in the analysis.

Stiffness in x-direction

Equivalent beam elements 5 and 6 contribute to stiffness in x-direction.

Equivalent beam elements 5 and 6 (ignoring flanges)

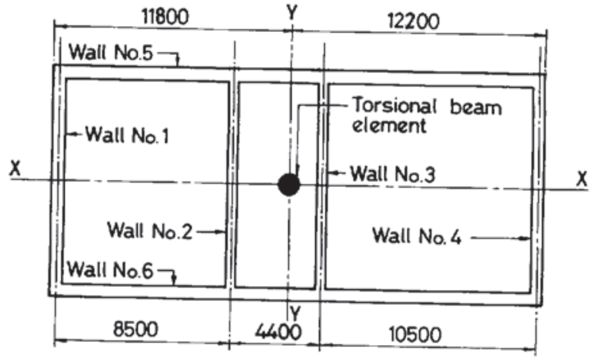
$$I_y = \frac{1}{12} \times 600 \times 24\,000^3 = 691.2\,\text{m}^4$$

Shear area $= 0.8 \times 600 \times 24\,000 = 11.52\,\text{m}^2$

Note: The moments of inertia and shear area of equivalent beam elements 5 and 6 about the x-axis will be ignored in the analysis.

Step 2 ***Find torsional stiffness of wall system***

Equivalent torsional rigidity element of the closed cell structure may be located at the centroid of the cell.



SK 8/43 Location of torsional vertical beam element.

By taking moments of areas about the left-hand edge of the cell,

$$x = \frac{0.6 \times 10 \times 0.3 + 0.4 \times 10 \times 8.8 + 0.4 \times 10 \times 13.2 + 0.6 \times 10 \times 23.7 + 2 \times 0.6 \times 24 \times 12}{6 + 4 + 4 + 6 + 2 \times 14.4}$$

$$= 11.8\,\text{m}$$

Areas of cells on centreline are as follows:

$$A_1 = 10.6 \times 8.5 = 90.1\,\text{m}^2$$
$$A_2 = 10.6 \times 4.4 = 46.6\,\text{m}^2$$
$$A_3 = 10.6 \times 10.5 = 111.3\,\text{m}^2$$

$$p_1 = \sum\left(\frac{\text{length of each arm of cell 1}}{\text{thickness of arm}}\right)$$
$$= \left(\frac{2 \times 8.5}{0.6}\right) + \left(\frac{10.6}{0.6}\right) + \left(\frac{10.6}{0.4}\right)$$
$$= 72.5$$

Similarly,

$$p_2 = \left(\frac{2 \times 4.4}{0.6}\right) + \left(\frac{2 \times 10.6}{0.4}\right)$$
$$= 67.7$$

$$p_3 = \left(\frac{2 \times 10.5}{0.6}\right) + \left(\frac{10.6}{0.6}\right) + \left(\frac{10.6}{0.4}\right)$$
$$= 79.2$$

$$p_{1.2} = \frac{10.6}{0.4} = 26.5$$

$$p_{2.3} = \frac{10.6}{0.4} = 26.5$$

Substituting $n = 3$ in the general equations in Section 8.1.1.5.3.

$$X_1'p_1 - X_2'p_{1,2} + 0 = A_1$$
$$-X_1'p_{1,2} + X_2'p_2 - X_3'p_{2,3} = A_2$$
$$0 - X_2'p_{2,3} + X_3'p_3 = A_3$$

or
$$72.5X_1' - 26.5X_2' + 0 = 90.1$$
$$-26.5X_1' + 67.7X_2' - 26.5X_3' = 46.6$$
$$0 - 26.5X_2' + 79.2X_3' = 111.3$$

Solving these equations:

$$X_1' = 2.11\,\text{m}^2 \qquad X_2' = 2.38\,\text{m}^2 \qquad X_3' = 2.2\,\text{m}^2$$

$$J = 4\sum_{1}^{n=3}(A_iX_i')$$
$$= 4(A_1X_1' + A_2X_2' + A_3X_3')$$
$$= 4(90.1 \times 2.11 + 46.6 \times 2.38 + 111.3 \times 2.2)$$
$$= 2183.5\,\text{m}^4$$

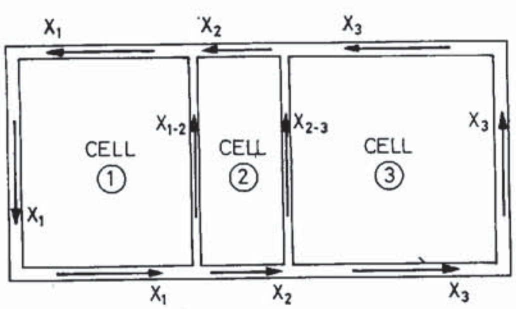
It is always useful to check at this stage the torsional rigidity of the outer cell, ignoring the internal dividing walls. This gives confidence in the numerical accuracy of the analysis.

For single outer cell:

$$J = \frac{4A^2}{\Sigma(B/h)}$$
$$= \frac{4 \times (23.4 \times 10.6)^2}{(2 \times 23.4/0.6) + (2 \times 10.6/0.6)}$$
$$= 2171\,\text{m}^4$$

This value is very close to the multiple cell rigidity.

Note: The torsional beam element to be used in the analysis will have negligible moments of inertia and shear area.



SK 8/44 Torsional shear flow diagram.

$$X_1 = \left(\frac{2T}{J}\right)X_1' = 2.18 \times 10^{-3}T\,\text{kN/m}$$

$$X_2 = \left(\frac{2T}{J}\right)X_2' = 2.18 \times 10^{-3}T\,\text{kN/m}$$

$$X_3 = \left(\frac{2T}{J}\right)X_3' = 2.02 \times 10^{-3} T \text{ kN/m}$$

Step 3 ***Carry out modelling for analysis***

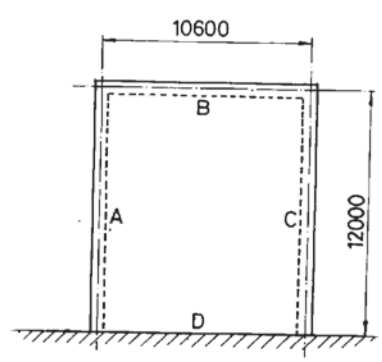
Follow recommendations in Section 8.1.2.1 and 8.1.2.2.

Step 4 ***Carry out global analysis***

The results of the analysis for different loadings are as follows:

Global torsion
= +50 000 kNm (clockwise) for horizontal load in y-direction
= +40 000 kNm (clockwise) for horizontal load in x-direction

Wall no.	Load case	N (kN)	M_I (kNm)	V_I (kN)	Local shear flow (kN/m)
	DL	3180	–		
1	LL	1325	–		
	WL(y)	–	+20 000	+1700	+97
	DL	980	–		
2	LL	615	–		
	WL(y)	–	+13 350	+1200	−12
	DL	980	–		
3	LL	615	–		
	WL(y)	–	+13 350	+1200	+8
	DL	3180	–		
4	LL	1325	–		
	WL(y)	–	+20 000	+1700	−101
	DL	7200	–		
5	LL	3125	–		
	WL(x)	–	+35 000	+3000	+87
	DL	7200	–		
6	LL	3125	–		
	WL(x)	–	+35 000	+3000	−87

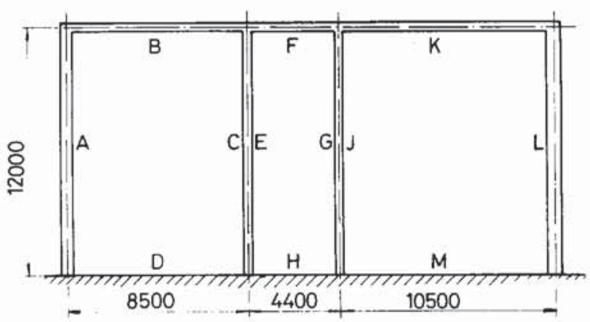


SK 8/45 Elevation – Wall 1.

Step 5 Carry out local analysis

Find out-of-plane internal forces in wall panels (follow Section 8.1.2.2). After analysis the following internal forces are reported:

Wall no.	Line	Load case	M_{OH} (kNm/m)	V_{OH} (kN/m)	M_{OV} (kNm/m)	V_{OV} (kN/m)
		DL	–	–	–	–
1	A	LL	–	–	–	–
		WL	–	–	28	26
		DL	20	3	–	–
	B	LL	10	1	–	–
		WL	32	30	–	–
		DL	–	–	–	–
	C	LL	–	–	–	–
		WL	–	–	28	26
		DL	10	3	–	–
	D	LL	5	1	–	–
		WL	32	30	–	–



SK 8/46 Elevation – Wall 5.

Wall 1 only will be designed as an example.
Wall 5 panels are shown in sketch to illustrate the location of lines where results should be available for out-of-plane bending.

Note: The example shown uses only one value of bending moment and shear per line of interest. In practice, more values along the line will have to be considered.

Step 6 Carry out combination of loading

Most computer programs used for the analysis will automatically carry out the combination according to principles described in Step 6 of Section 8.2. Reproduced below is the result of one combination of Wall 1.

Load case $LC_3 = 1.4DL + 1.4WL$

Wall 1 subject to WL (y-direction)
In-plane forces (see Step 4 in Section 8.2):

$N = 4452\,\text{kN}$
$M_I = 28\,000\,\text{kNm}$
$V_I = 2380\,\text{kN}$
$Q_I = 1440\,\text{kN}\ (97 \times 10.6 \times 1.4)$

Out-of-plane forces (see Step 5 in Section 8.2):

At line D (WL in *y*-direction),

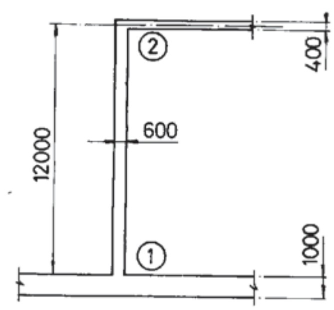
$M_{OH} = 14\,\text{kNm/m}$ (dead load × 1.4)
$V_{OH} = 42\,\text{kN/m}$ (DL × 1.4)

On flanges (part of Wall 5 and 6),

$M_{OH} = 39\,\text{kNm/m}$ (WL in *y*-direction)
$V_{OH} = 36\,\text{kN/m}$ (WL in *y*-direction)

Step 7 Check slenderness of wall
Follow Section 8.1.1.2.1.

Type of wall = unbraced, reinforced in the in-plane direction



SK 8/47 Section through Wall 1.

$f_{cu} = 30\,\text{N/mm}^2 \qquad f_y = 460\,\text{N/mm}^2$

$H_e = \beta\, H_o$

H_o = clear height $= 12.0\,\text{m}$

Monolithic construction at top and bottom of wall.
Assume thickness of slab at top is 400 mm.
End conditions are 1 at bottom and 2 at top.

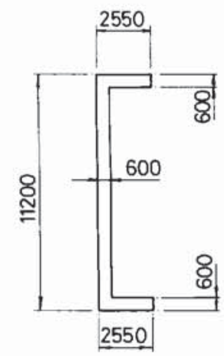
$\therefore\ \beta = 1.3$

$H_e = 1.3 \times 12\,000 = 15\,600\,\text{mm}$

$$\frac{H_e}{h} = \frac{15\,600}{600} = 26 > 10 < 30 \quad \text{(limit for unbraced reinforced wall)}$$

Design as slender wall.

Step 8 ***Find effective width of flanges***



SK 8/48 Plan of Wall 1 showing effective flange widths.

Follow Section 8.1.1.3.
Assume the shear wall behaves as a cantilever.

$$b = \frac{8500}{2} = 4250 \qquad H = 12\,000$$

$$\frac{b}{H} = 0.35$$

$\psi = 0.53$ for loading at top of wall

$$b_e = \psi\, b = 0.53 \times 4250 = 2250\,\text{mm}$$

Step 9 ***Find additional out-of-plane moments***
Wall is assumed braced in the out-of-plane direction.

$$a_u = \beta\, Kh,\ H_e = \beta\, H_o = 0.8 \times 12\,000 = 9600$$

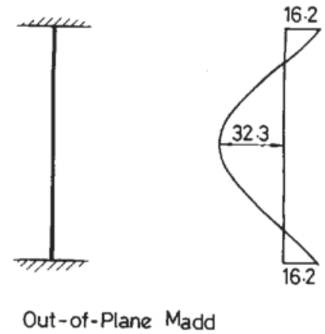
Assume $K = 1$ for conservatism.

$$\beta = \frac{1}{2000}\left(\frac{H_e}{h}\right)^2 = \frac{256}{2000} = 0.128$$

$$a_u = 0.128h = 76.8\,\text{mm}$$

$$\begin{aligned} M_{add} &= Na_u \quad \text{(out-of-plane)} \\ &= 4452 \times 0.0768 \\ &= 342\,\text{kNm} \\ &= \frac{342}{10.6} = 32.3\,\text{kNm/m} \end{aligned}$$

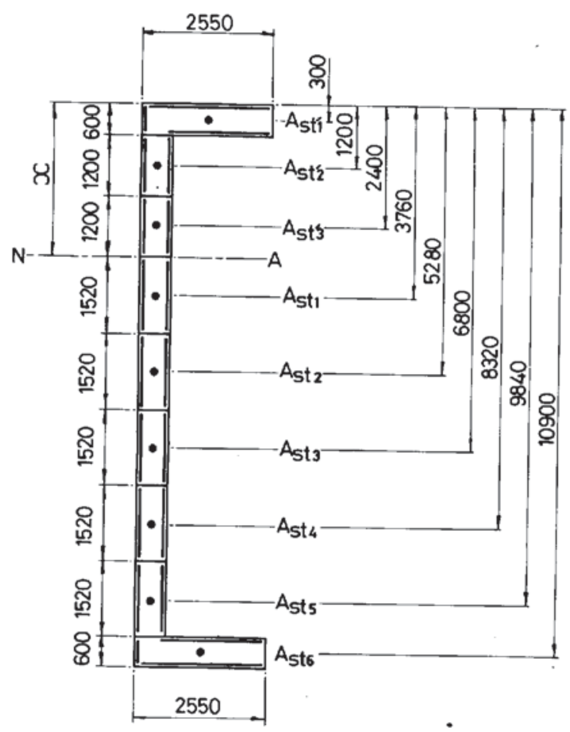
Step 10 ***Design stocky braced reinforced wall***
Not applicable.



SK 8/49 Moments due to slenderness.

Step 11 ***Determine cover to reinforcement***
Maximum size of aggregate = 20 mm
Condition of exposure = mild
Grade of concrete = C30
Minimum cement content = 275 kg/m^3
Nominal cover = 25 mm

Step 12 ***Design of reinforced wall – rigorous method***



SK 8/50 Elastic stress analysis of Wall 1.

(1) Assume 0.40% reinforcement in wall.

$$\text{Reinforcement per metre length} = \frac{600 \times 1000 \times 0.40}{100} = 2400\,\text{mm}^2/\text{m}$$

Use $1200\,\text{mm}^2$ on each face per metre length.

(2) Assume a value of x for depth of neutral axis from compression face.

$x = 3000\,\text{mm}$ assumed.

(3) Divide compression zone into convenient layers of concrete.
(4) Divide tension zone into convenient layers of steel.
(5) Find the following in completing the table:

$$S = A_c + (m - 1)A'_{st}$$

$$C_1 = (x - a_c)S$$
$$C_2 = (x - a_c)a_cS$$
$$C_3 = (a_t - x)A_{st}$$
$$C_4 = (a_t - x)a_tA_{st}$$

Number	A_c ($\times 10^6$)	A'_{st}	S ($\times 10^6$)	a_c	A_{st}	a_t	C_1 ($\times 10^9$)	C_2 ($\times 10^{12}$)	C_3 ($\times 10^6$)	C_4 ($\times 10^{10}$)
1	1.53	6120	1.62	300	3648	3760	4.374	1.312	2.772	1.042
2	0.72	2880	0.76	1200	3648	5280	1.368	1.642	8.317	4.392
3	0.72	2880	0.76	2400	3648	6800	0.456	1.094	13.862	9.426
4	—	—	—	—	3648	8320	—	—	19.407	16.147
5	—	—	—	—	3648	9840	—	—	24.952	24.553
6	—	—	—	—	6120	10900	—	—	48.348	52.699
Totals	2.97	11880	3.14		24360		6.198	4.048	117.658	108.259

$$A_c = \frac{\Sigma C_2}{\Sigma C_1} = \frac{4.048 \times 10^{12}}{6.198 \times 10^9} = 653\,\text{mm}$$

$$A_T = \frac{\Sigma C_4}{\Sigma C_3} = 9201\,\text{mm}$$

$$\bar{x} = \frac{m\Sigma A_{st}a_t + \Sigma Sa_c}{m\Sigma A_{st} + \Sigma S} = 1735\,\text{mm}$$

$$e = \frac{M}{N} = \frac{28\,000 \times 10^3}{4452} = 6289\,\text{mm}$$

$$f_c = \frac{Nx(e + A_T - \bar{x})}{(A_T - A_c)\,\Sigma\,(x - a_c)S}$$
$$= \frac{4452 \times 10^3 \times 3000 \times (6289 + 9201 - 1735)}{(9201 - 653) \times 6.198 \times 10^9}$$
$$= 3.467\,\text{N/mm}^2$$

$$f_{st} = \left[\frac{d - x}{\Sigma\,(a_t - x)A_{st}}\right]\left[\left(\frac{f_c}{x}\right)\sum\,(x - a_c)S - N\right]$$
$$= \left(\frac{11\,100 - 3000}{117.658 \times 10^6}\right) \times \left[\left(\frac{3.467}{3000}\right) \times 6.198 \times 10^9 - 4452 \times 10^3\right]$$
$$= 186.6\,\text{N/mm}^2$$

(6) Check $x = \dfrac{d}{1 + \dfrac{f_{st}}{mf_c}} = 2419\,\text{mm}$

Second approximation for x is halfway between first approximation and the check result.

Assume $x = 2700\,\text{mm}$

After carrying out the same tabular exercise as before it is found that:

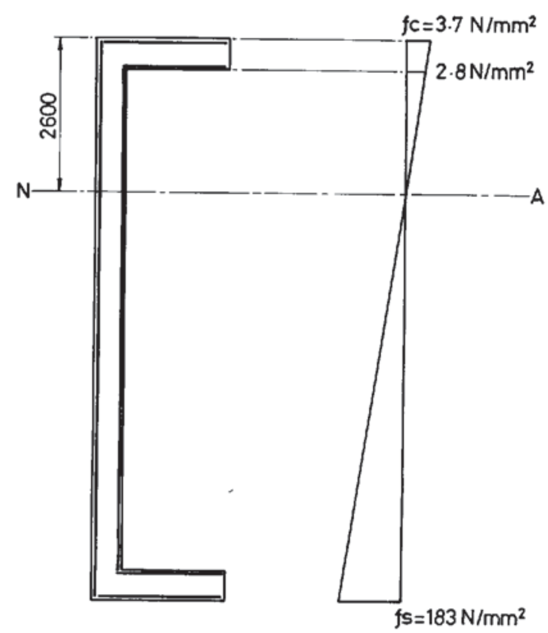
$f_c = 3.66\,\text{N/mm}^2$
$f_{st} = 182.4\,\text{N/mm}^2$

Check $x = 2570\,\text{mm}$.

No further iteration is necessary.

Check reinforcement in compression flange due to out-of-plane bending

Average compressive stress in flange $= \frac{1}{2}(3.7 + 2.8) = 3.25\,\text{N/mm}^2$



SK 8/51 Elastic analysis – stress diagram.

Average compressive stress in reinforcement in flange $= 3.25 \times 15$ for $m = 15$
$= 48.75\,\text{N/mm}^2$

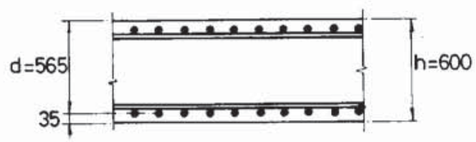
Over a unit length of wall,

compressive force, $N = 3.25 \times 600 = 1950\,\text{kN}$

Out-of-plane bending moment due to $DL + WL(y)$ + additional moment due to slenderness

$= 39 + \dfrac{32.3}{2}$ (see Step 9)

$= 55.2\,\text{kNm/m}$



SK 8/52 Section through wall for out-of-plane analysis.

Material strengths chosen:

$f_{cu} = 30\,\text{N/mm}^2 \quad f_y = 460\,\text{N/mm}^2$

$k = \dfrac{d}{h} = \dfrac{565}{600} = 0.95 \quad e = \dfrac{M}{N} = 0.028\,\text{m}$

See Table 11.8.

$\dfrac{e}{h} = 0.047$

For $p = 0.4$, $\dfrac{N}{bh}$ from chart is $13.53 > 3.25\,\text{N/mm}^2$

$\therefore$ Nominal steel is required as per chart.

Check reinforcement in tension flange for out-of-plane bending

Maximum tensile stress in bar due to in-plane bending moment = $183\,\text{N/mm}^2$

Maximum allowable ultimate tensile stress in bars $= 0.87f_y$
$= 0.87 \times 460$
$= 400\,\text{N/mm}^2$

Available tensile force in bars per metre length of wall per face of wall
$= (400 - 183) \times 1200$ (area on each face)
$= 260.4\,\text{kN/m}$

Maximum out-of-plane bending moment $= 55.2\,\text{kNm/m}$

$$K = \frac{M}{f_{cu}bd^2}$$
$$= \frac{55.2 \times 10^6}{30 \times 1000 \times 565^2}$$
$$= 5.76 \times 10^{-3}$$

$$z = d\left[0.5 + \sqrt{\left(0.25 - \frac{K}{0.9}\right)}\right] \leq 0.95d$$
$$= 0.95d = 537\,\text{mm}$$

$$\text{Required tensile force in bars} = \frac{M}{z} = \frac{55.2 \times 10^3}{537} = 102.8\,\text{kN/m}$$

This is less than 260.4 kN/m available. Hence, no additional reinforcement is required in tension flange.

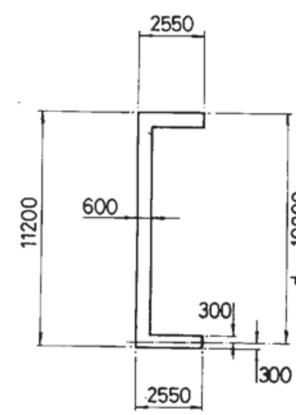
Step 13 Design of reinforced wall – simple method

Not required. The design principle is exactly similar to beam design and has not been illustrated.

Step 14 Design of short and squat cantilever wall – deep beam approach

Not required, because $H/L > 1$.

Step 15 Check shear



SK 8/53 Shear check of Wall 1.

A_{si} = available tension reinforcement below neutral axis in web ignoring flange
$= (11.2 - 2.6) \times 2400$
$= 20\,640\,\text{m}^2$

$$d_i = 11200 - 300 = 10\,900\,\text{mm (approx.)}$$

$$p_i = \frac{100A_{si}}{h_w d_i} = \frac{100 \times 20\,640}{600 \times 10\,900} = 0.32\%$$

$f_{cu} = 30\,\text{N/mm}^2$

From Figs 11.2 to 11.5,

$v_{ci} = 0.47\,\text{N/mm}^2$

V_i = combined in-plane shear
$= V_I + Q_I = 2380 + 1440 = 3820\,\text{kN}$

$$v_i = \frac{V_i}{h_w d_i} = 0.58\,\text{N/mm}^2$$

A_{so} = available tension reinforcement for out-of-plane bending
$= 1200\,\text{mm}^2/\text{m}$ (each face)

d_o = effective depth in out-of-plane direction = 565 mm

$$p_o = \frac{100A_{so}}{bd_o} = \frac{100 \times 1200}{1000 \times 565} = 0.21\%$$

From Figs 11.2 to 11.5,

$v_{co} = 0.4\,\text{N/mm}^2$

V_{OH} = out-of-plane shear coacting with V_i
$= 4.2\,\text{kN/mm}$ on the web

$$v_{oh} = \frac{V_{OH}}{bd_o} = 0.007\,\text{N/mm}^2$$

$$\frac{v_i}{v_{ci}} + \frac{v_{oh}}{v_{co}} = \left(\frac{0.58}{0.47}\right) + \left(\frac{0.007}{0.4}\right) = 1.25 > 1$$

Shear reinforcement is necessary for in-plane shear.

Note: Increase of design concrete shear stress due to presence of axial load has been ignored in these calculations but may be allowed as per formula on page 160.

Step 16 Calculate shear reinforcement

Case 1: $v_{oh} < v_{co}$

$$v_i' = \left(1 - \frac{v_{oh}}{v_{co}}\right) v_{ci}$$
$$= \left(1 - \frac{0.007}{0.4}\right) \times 0.47$$
$$= 0.46\,\text{N/mm}^2$$

$V_i' = v_i' h_w d_i = 3008.4\,\text{kN}$

$V_i - V_i' = 3820 - 3008.4 = 811.6\,\text{kN}$

Shear reinforcement is required to resist 811.6 kN.

$$\frac{A_h}{S_h} = \frac{V_{si}}{0.87 f_y d_i}$$
$$= \frac{811.6 \times 10^3}{0.87 \times 460 \times 10\,900}$$
$$= 0.19$$

If $S_h = 300$, then $A_h = 300 \times 0.19 = 57\,\text{mm}^2$ which is $29\,\text{mm}^2$ of horizontal bar on each face at 300 mm centres, or, $97\,\text{mm}^2$ per metre on each face.

$$\frac{A_v}{S_v} = \frac{A_h}{S_h} = 0.19 \qquad \text{for } f_y = 460\,\text{N/mm}^2$$

Vertical shear reinforcement additional to vertical bars provided for bending is required if available vertical bars have no residual capacity.
In the web $2400\,\text{mm}^2/\text{m}$ vertical bars are available at a maximum average stress level of, say, $160\,\text{N/mm}^2$ (see Step 12). Hence residual capacity available in vertical bars in web $= 0.87 \times 460 - 160 = 240\,\text{N/mm}^2$

Modified A_v/S_v to take into account the residual capacity

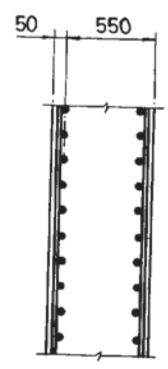
$$= 0.19 \times 0.87 \times \frac{460}{240} = 0.32$$

A_v for shear required per metre length of wall $= 320\,\text{mm}^2$ (modified)

Available vertical bars $= 2400\,\text{mm}^2/\text{m}$ in web

Hence no additional vertical bars are necessary to resist shear in web.
No shear reinforcement is required in out-of-plane direction.

Step 17 ***Check out-of-plane bending about vertical plane***



SK 8/54 Out-of-plane bending about vertical plane.

$$M_{OV} = 1.4 \times 28 \qquad \text{(see Step 5)}$$
$$= 39.2\,\text{kNm/m}$$

$$V_{OV} = 1.4 \times 26$$
$$= 36.4\,\text{kN/m}$$

$$K = \frac{M}{f_{cu}bd^2} = \frac{39.2 \times 10^6}{30 \times 1000 \times 550^2} = 4.3 \times 10^{-3}$$

$$z = d\left[0.5 + \sqrt{\left(0.25 - \frac{K}{0.9}\right)}\right] \leq 0.95d$$

$$= 0{,}95d = 522\,\text{mm}$$

$$A_s = \frac{M}{0.87f_y z} = \frac{39.2 \times 10^6}{0.87 \times 460 \times 522} = 187.6\,\text{mm}^2$$

Add to this reinforcement the horizontal reinforcement required in Step 16 for in-plane shear.

Total horizontal reinforcement required on each face (assuming the load WL is reversible in direction)
$= 187.6 + 97 = 284.6\,\text{mm}^2/\text{m}$

$$v_{ov} = \frac{V_{ov}}{bd} = \frac{36.4 \times 10^3}{1000 \times 550} = 0.07\,\text{N/mm}^2$$

Shear stress is negligible.

***Step 18* Design of plain walls**
Not required.

***Step 19* Shear check of plain walls**
Not required.

***Step 20* Check minimum reinforcement**
Minimum compression vertical reinforcement in wall = 0.4%
($f_y = 460\,\text{N/mm}^2$)
This has been provided.

Minimum horizontal tension reinforcement on each face
$= 0.13\% \quad (f_y = 460\,\text{N/mm}^2)$

$$= 0.13 \times 1000 \times \frac{600}{100}$$

$= 780\,\text{mm}^2/\text{m}$ on each face

This amount is greater than horizontal reinforcement found in Step 17. This reinforcement will be adopted.
Minimum anti-crack reinforcement is 0.25% in both directions on each face. This has been provided.

***Step 21* Check maximum reinforcement**
Not required.

***Step 22* Check containment of wall reinforcement**
Vertical reinforcement is less than 2% of gross concrete area.

Hence requirement is to provide horizontal reinforcement equal to 0.2% of gross cross-sectional area. This is provided.

Vertical bar diameter = 20 mm

Horizontal bar diameter = 10 mm $> \frac{1}{4}$ (20 mm)

Step 23 Check early thermal cracking

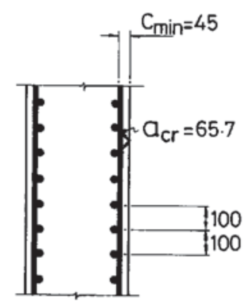
Crack width limitation = 0.3 mm

(see Step 14 of Section 3.3).
Assume $R = 0.8$ at base.

$T_1 = 32°C$

$$\varepsilon_r = 0.8 T_1 \alpha R$$
$$= 0.8 \times 32 \times 12 \times 10^{-6} \times 0.8 = 2.46 \times 10^{-4}$$

Check horizontal bars for vertical cracks



SK 8/55 Crack width for horizontal bars 10 mm @ 100 c/c.

Assume 10 mm diameter bars at 100 mm centres (785 mm^2/m).

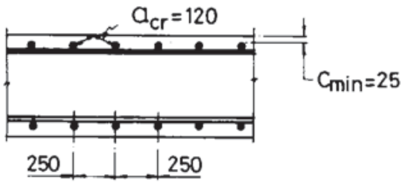
$a_{cr} = 65.7$ mm $(1.414 \times 50 - 5 = 65.7)$

take $x = h/2$

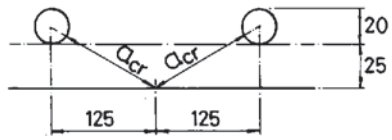
$$W_{max} = \frac{3a_{cr}\ \varepsilon_r}{1 + \dfrac{2(a_{cr} - C_{min})}{h - x}}$$
$$= \frac{3 \times 65.7 \times 2.46 \times 10^{-4}}{1 + \dfrac{2(65.7 - 45)}{300}}$$
$$= 0.04\text{ mm} < 0.3\text{ mm} \quad \text{OK}$$

Check vertical bars for horizontal cracks

Assume 20 mm diameter bars at 250 mm centres vertically (1256 mm^2/m each face).



SK 8/56 Crack width for vertical bars 20 mm @ 250 c/c.



SK 8/57 Sketch to find a_{cr}.

$$a_{cr} = 120\,\text{mm}$$

$$W_{max} = \frac{3 \times 120 \times 2.46 \times 10^{-4}}{1 + \dfrac{2(120 - 25)}{300}}$$

$$= 0.05\,\text{mm} < 0.3\,\text{mm} \quad \text{OK}$$

Step 24 Clear spacings of bars in tension

Reinforcement provided is 20 mm diameter at 250 mm centres both faces vertically and 10 mm diameter at 100 mm centres both faces horizontally. These spacings satisfy the requirements according to Step 13 of Section 3.3.

Step 25 Connections

Follow Chapter 10.