Chapter 2

Design of Reinforced Concrete Beams

2.0 NOTATION

\( a' \)  Compression face to point on surface of concrete where crack width is calculated
\( a_{bc} \) Centre-to-centre distance between bars or groups of bars
\( a_{ct} \) Point on surface of concrete to nearest face of a bar
\( A_c \) Gross area of concrete in a section
\( A_s \) Area of steel in tension
\( A_s' \) Area of steel in compression
\( A_{sv} \) Area of steel in vertical links
\( b \) Width of reinforced concrete section
\( b_c \) Breadth of compression face of beam mid-way between restraints
\( b_t \) Width of section at centroid of tensile steel
\( b_w \) Average web width
\( c \) Coefficient of torsional stiffness
\( c_{min} \) Minimum cover to tensile reinforcement
\( C \) Torsional stiffness
\( d \) Effective depth of tensile reinforcement
\( d' \) Effective depth of compressive reinforcement
\( d_t \) From tension face of concrete section to centre of tensile reinforcement
\( E_c \) Modulus of elasticity of concrete
\( E_s \) Modulus of elasticity of steel
\( f_s \) Service stress in steel reinforcement
\( f_y \) Characteristic yield strength of steel
\( f_y' \) Revised compressive stress in steel taking into account depth of neutral axis
\( f_{cu} \) Characteristic cube strength of concrete at 28 days
\( f_{yy} \) Characteristic yield strength of reinforcement used as links
\( F \) Coefficient for calculation of cracked section moment of inertia
\( F_{bt} \) Tensile force in a bar at start of a bend
\( G \) Shear modulus
\( h \) Overall depth of a concrete section
\( h_t \) Thickness of flange in a T-beam
\( h_{max} \) Maximum overall dimension of a rectangular concrete beam
\( h_{min} \) Minimum overall dimension of a rectangular concrete beam
\( I \) Moment of inertia
\( l \) Clear span or span face-to-face of support
Reinforced Concrete

\( l_e \)  Effective span
\( l_o \)  Centre-to-centre distance between supports
\( m \)  modular ratio = \( E_s/E_c \)
\( M \)  Applied bending moment
\( M_d \)  Design bending moment modified to account for axial load
\( M_r \)  Moment of resistance of concrete in flanged beams
\( N \)  Axial load
\( p \)  Percentage of tensile reinforcement
\( p' \)  Percentage of compressive reinforcement
\( r \)  Internal radius of a bend in a bar
\( S_b \)  Spacings of bent bars used as shear reinforcement
\( S_c \)  Spacing of vertical links
\( T \)  Applied torsion
\( T_v \)  Proportion of total torsion carried by each rectangle of an I-, T- or L-section
\( v \)  Shear stress in concrete (N/mm²)
\( v_c \)  Design concrete shear stress (N/mm²)
\( v_t \)  Shear stress in concrete due to torsion (N/mm²)
\( v_{tu} \)  Ultimate permissible torsional shear stress (N/mm²)
\( v_{t,\text{min}} \)  Design concrete torsional shear stress (N/mm²)
\( V \)  Shear force in concrete section
\( V_b \)  Shear force carried by bent bars
\( V_c \)  Shear force capacity of concrete section
\( V_s \)  Shear force carried by vertical links
\( V_{\text{max}} \)  Ultimate maximum shear forces allowed on section
\( V_{\text{nom}} \)  Shear force capacity of concrete section with minimum vertical links
\( V_{\text{concrete}} \)  Design shear resistance of concrete
\( W_{\text{max}} \)  Maximum crack width (mm)
\( x \)  Depth of neutral axis from compression face
\( x_i \)  Centre-to-centre of two external vertical legs of a link
\( y_l \)  Centre-to-centre of two external horizontal legs of a link
\( z \)  Depth of lever arm

\( \alpha \)  Angle of inclination to horizontal of shear reinforcement
\( \beta \)  Angle of inclination to horizontal of concrete strut in truss analogy
\( \beta_p \)  Ratio of redistributed moment over elastic analysis moment
\( \beta_T \)  Factor governing moment of resistance of concrete T-section
\( \gamma_m \)  Material factor
\( \varepsilon_h \)  Calculated strain in concrete at depth \( h \)
\( \varepsilon_m \)  Strain with stiffening effect corrected
\( \varepsilon_s \)  Strain at centre of steel reinforcement
\( \varepsilon_y \)  Yield strain in steel reinforcement
\( \varepsilon_{sh} \)  Strain at centre of compressive reinforcement
\( \varepsilon_{shh} \)  Strain at depth \( h \) corrected for stiffening effect
\( \varepsilon_l \)  Calculated strain in concrete ignoring stiffening effect
\( \mu \)  Poisson’s ratio
\( \phi \)  Diameter of a reinforcing bar or equivalent diameter of a group of bars
2.1 ANALYSIS OF BEAMS

2.1.1 Effective spans

Simply supported or encastré \( l_e = \) smaller of \((l + d)\) or \(l_o\)

Continuous \( l_e = l_o \)

Cantilever \( l_e = l + \frac{d}{2} \)

where \( l_o = \) centre-to-centre distance between supports
\( l_e = \) effective span
\( l = \) clear span or span to face of support
\( d = \) effective depth of tension reinforcement.

2.1.2 Effective width of compression flange

Simply supported T-beam \( b = \frac{l_e}{5} + b_w \)

Simply supported L-beam \( b = \frac{l_e}{10} + b_w \)

Continuous or encastré T-beams \( b = \frac{l_e}{7.14} + b_w \)

Continuous or encastré L-beams \( b = \frac{l_e}{14.29} + b_w \)
where \( b \) = effective width of compression flange
\[ \frac{b}{w} = \text{average width of web}. \]

**Note:** Use actual \( b \) if it is less than the calculated \( b \) using the above formulae.
A typical example may be a precast T-beam.

### 2.1.3 Moment of inertia

**Method 1** Gross concrete section only
Find moment of inertia of gross concrete section – see Table 11.2.

**Method 2** Uncracked transformed concrete
If reinforcement quantities are known, find moment of inertia of transformed concrete section using Table 11.2.

**Method 3** Average of gross concrete section and cracked section

\[
I = 0.5 \left( \frac{1}{12} bh^3 + Fbh^3 \right)
\]

where \( I \) = moment of inertia of rectangular concrete section
\( b \) = width of rectangular concrete section
\( h \) = overall depth of rectangular concrete section
\( F \) = factor – see Fig. 11.1 for values of \( F \).

\[
p = 100 \frac{A_s}{bd}
\]

where \( A_s \) = area of tensile reinforcement
\( d \) = effective depth to tensile reinforcement.

\[
p' = 100 \frac{A'_s}{bd}
\]

where \( A'\_s \) = area of compressive reinforcement.
Design of Reinforced Concrete Beams

\[ m = \text{modular ratio} = \frac{E_s}{E_c} \]

The graphs in Fig. 11.1 have been drawn for \( p' = 0 \) and \( p = p' \). Intermediate values may be interpolated.

**Note:** The preferred method is Method 3 for rectangular sections. Where reinforcement quantities are not known, an assumption may be made of the percentage of reinforcement.

T-beams and L-beams in a frame or continuous beam structure should be treated as rectangular beams for the purpose of determining moment of inertia. The width of the beam will be taken equal to \( b_w \).

### 2.1.4 Modulus of elasticity

Modulus of elasticity of reinforcement steel

\[ E_s = 200 \text{kN/mm}^2 \]

Modulus of elasticity of concrete, \( E_c \), for short-term and long-term loadings is given in Table 2.1.

**Table 2.1 Modulus of elasticity of concrete: short-term and long-term loading.**

<table>
<thead>
<tr>
<th>( f_{cu} ) (N/mm(^2))</th>
<th>Short-term loading, ( E_c ) (kN/mm(^2))</th>
<th>Long-term loading, ( E_c ) (kN/mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>12.5</td>
</tr>
<tr>
<td>30</td>
<td>26</td>
<td>13</td>
</tr>
<tr>
<td>40</td>
<td>28</td>
<td>14</td>
</tr>
<tr>
<td>50</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>60</td>
<td>32</td>
<td>16</td>
</tr>
</tbody>
</table>

**Note:** Wind load is short-term loading and dead load is long-term loading.

### 2.1.5 Torsional stiffness

For a rectangular section the torsional stiffness, \( C \), is given by

\[ C = \frac{c h_{\min}^3 h_{\max}}{} \]

where \( c = \text{coefficient from Table 2.2} \)

\( h_{\max} = \text{maximum overall dimension of rectangular section} \)

\( h_{\min} = \text{minimum overall dimension of rectangular section} \).
The torsional stiffness of a non-rectangular section may be obtained by dividing the section into a series of rectangles and summing the torsional stiffness of these rectangles.

Table 2.2 Values of coefficient \( c \).

<table>
<thead>
<tr>
<th>( \frac{h_{\text{max}}}{h_{\text{min}}} )</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>0.14</td>
<td>0.20</td>
<td>0.23</td>
<td>0.26</td>
<td>0.29</td>
<td>0.31</td>
</tr>
</tbody>
</table>

The coefficient \( c \) is given by the following formula:

\[
c = \frac{1}{16} \left[ \frac{16}{3} - 3.36k \left( 1 - \frac{k^4}{12} \right) \right]
\]

where \( k = \frac{h_{\text{min}}}{h_{\text{max}}} \).

2.1.6 Shear modulus

Shear modulus, \( G \), is given by

\[
G = \frac{E}{2} \left( 1 + \mu \right) = 0.42E_c \quad \text{for concrete}
\]

where \( \mu = \text{Poisson’s ratio} \).

Note: In normal slab and beam or framed construction, torsional rigidity of RC beams may be ignored in the analysis and the torsional stiffness may be given a very small value in the computer analysis. Torsional rigidity becomes important only where torsion is relied on to carry the load, as in curved beams.
2.1.7 Poisson's ratio

Poisson's ratio for concrete = 0.2

2.1.8 Shear area

Shear area of concrete = $0.8A_c$

where $A_c$ = gross cross-sectional area of concrete.

**Note:** The shear area of concrete is entered as input to some computer programs when the analysis is required to take into account the deformations due to shear.

2.1.9 Thermal strain

The coefficients of thermal expansion are given in Table 2.3 for different types of aggregate used.

<table>
<thead>
<tr>
<th>Aggregate type</th>
<th>Coefficient ($\times 10^{-6} / ^\circ C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flint, Quartzite</td>
<td>12</td>
</tr>
<tr>
<td>Granite, Basalt</td>
<td>10</td>
</tr>
<tr>
<td>Limestone</td>
<td>8</td>
</tr>
</tbody>
</table>

**Note:** Normally for ultimate limit state no specific calculations are necessary for thermal loads. Thermal calculations should be produced for structures in contact with hot gases or liquid.

2.2 LOAD COMBINATIONS

2.2.1 General rules

The following load combinations and partial load factors should be used in carrying out the analysis of beams:

$LC_1$: $1.4 \, DL + 1.6 \, LL + 1.4 \, EP + 1.4 \, WP$

$LC_2$: $1.0 \, DL + 1.4 \, EP + 1.4 \, WP$

$LC_3$: $1.4 \, DL + 1.4 \, WL + 1.4 \, EP + 1.4 \, WP$

$LC_4$: $1.0 \, DL + 1.4 \, WL + 1.4 \, EP + 1.4 \, WP$

$LC_5$: $1.2 \, DL + 1.2 \, LL + 1.2 \, WL + 1.2 \, EP + 1.2 \, WP$

**Note:** Load combinations $LC_2$ and $LC_4$ should be considered when the effects of dead load and live load are beneficial.
where  $DL$ = dead load  \\
$LL$ = live load or imposed load  \\
$WL$ = wind load  \\
$WP$ = water pressure  \\
$EP$ = earth pressure.

The general principle of load combination is to leave out the loads which have beneficial effect. If the load is of a permanent nature, like dead load, earth load or water load, use the partial load factor of 1 for that load which produces a beneficial rather than adverse effect. This rule of combination will be used for design as well as for the check of stability of a structure.

2.2.2 Rules of load combination for continuous beams

- **LC$_{1}$ ON ALL SPANS**
- **LC$_{1}$ ALTERNATE WITH LC$_{2}$ FOR MAXIMUM MIDSPAN MOMENT**
- **LC$_{1}$ ON ADJACENT SPANS ALTERNATE WITH LC$_{2}$ FOR MAXIMUM SUPPORT MOMENT**

SK 2/7 Continuous beam loading sequences.

(1) Load all spans with $LC_{1}$.
(2) Load alternate spans with $LC_{1}$ and other spans with $LC_{2}$.
(3) Load beam in the repeated sequence of two adjacent spans loaded with $LC_{1}$ and one span loaded with $LC_{2}$. This sequence gives the maximum support moment between adjacent spans. This is not a normal requirement, as per clause 3.2.1.2.2 of BS 8110: Part 1: 1985.[1]
2.2.3 Redistribution of moments

2.2.3.1 Continuous beams

Usually 10% redistribution of moments may be allowed from those obtained by elastic analysis. Redraw bending moment diagram with redistributed moments. Calculate revised shear. Reduction of support moment means a corresponding increase in span moment. For structural frames over four stories high providing lateral stability, the redistribution of moments should not exceed 10%. Resistance moment at any section must be at least 70% of moment at that section obtained by elastic analysis.

2.2.3.2 Frame structures

No reduction or redistribution of moments is allowed from the columns.

2.2.3.3 Continuous one-way spanning slab panels

Usually 10% redistribution of moments may be allowed from those obtained by elastic analysis.

2.2.4 Exceptional loads

Exceptional loads may be any of the following.

1. Accidental loads of very low probability properly quantified. The definition of low probability may vary from project to project and will be agreed with the client.
2. Probable misuse and its effect accurately quantified.
3. Once in a lifetime very short-term loads which are accurately quantified.

Note: With exceptional loads some rectification of local damage after the incident may be necessary.

Load combination to be considered:

\[ LC_0 = 1.05DL + 1.05LL_1 + 1.05EL + 1.05WL_1 \]

where \( DL = \) full expected dead load
\( LL_1 = \) full expected live load if this is a storage building, otherwise, one-third of expected maximum live load
\( EL = \) exceptional load
\( WL_1 = \) one-third of expected maximum wind load.
2.3 STEP-BY-STEP DESIGN PROCEDURE FOR BEAMS

Step 1 Analysis
Carry out analysis – follow Section 2.1.

Step 2 Moment envelope

![Moment envelope diagram]

SK 2/9 Typical moment envelope of a continuous beam.

Draw maximum–minimum ultimate load bending moment envelope after redistribution.

Step 3 Shear envelope

![Shear envelope diagram]

SK 2/10 Typical shear envelope of a continuous beam.

Draw maximum–minimum ultimate load shear force envelope after redistribution.

Step 4 Axial loads
Determine coacting axial loads with maximum and minimum bending moments respectively. Ignore axial load if less than 0.1f_{cu}bh.

Step 5 Torsions
Determine coacting torsions with maximum and minimum shear forces respectively.
SK 2/11 Dimensions to compute axial load in beam.

**Step 6  Cover to reinforcement**
Determine cover required to reinforcement as per Tables 11.6 and 11.7.
Find effective depth $d$, assuming reinforcement diameter.

SK 2/12 Rectangular beam — effective depth.

**Step 7  Effective span**
Determine effective span — see Section 2.1.1.

**Step 8  Effective width of compression flange**
Determine effective width of compression flange — see Section 2.1.2.

**Step 9  Slenderness ratio**

SK 2/13 Simply supported and continuous beams.

SK 2/14 Cantilever beam.

Check slenderness of beam as per clause 3.4.1.6 of BS8110: Part 1: 1985.[1]

For simply supported and continuous beams,

\[
l < 60b_c \quad \text{or} \quad 250 \frac{b_c^2}{d}
\]

For cantilever beams,

\[
l < 25b_c \quad \text{or} \quad 100 \frac{b_c^2}{d}
\]
where \( d = \sqrt{\left( \frac{M}{0.156f_{cu}b} \right)} \) or actual \( d \), whichever is lesser

\( b \) = effective width of the compression flange
\( M \) = design ultimate moment.

**Step 10 Design for moment – rectangular beam**
Select critical sections on beam for bending moment. Find the following parameters at all critical sections, for rectangular beams and flanged beams.

\( M_d = M + N (0.5h - d_1) \) for \( N \leq 0.1f_{cu}bd \)

**Note:** For \( N > 0.1f_{cu}bd \), design as column (see Chapter 4). \( M_d \) may also be taken equal to \( M \) where \( N \leq 0.1f_{cu}bd \) and \( N \) may be totally ignored. (Sign convention: \( N \) is +ve for compression.)

\[ K = \frac{M_d}{f_{cu}bd^2} \]
\[ z = d \left[ 0.5 + \sqrt{\left( 0.25 - \frac{K}{0.9} \right)} \right] \leq 0.95d \]
\[ x = \frac{d - z}{0.45} \]
\[ A_s = \frac{M_d}{0.87f_yz} - \frac{N}{0.87f_y} \]
\[ K' = 0.156 \text{ when redistribution does not exceed 10\%} \]
\[ K' = 0.402(\beta_b - 0.4) - 0.18(\beta_b - 0.4)^2 \text{ when redistribution exceeds 10\%} \]
\[ \beta_b = \frac{M'^2}{M} < 0.9 \]
where \( M^2 \) = moment after redistribution; \( M' \) = moment before redistribution

When \( K > K' \),
\[
z = d \left[ 0.5 + \sqrt{\left( 0.25 - \frac{K'}{0.9} \right)} \right]
\]
\[
x = \frac{d - z}{0.45} \leq 0.5d
\]
\[
A'_s = \frac{(K - K')f_{cu}bd^2}{0.87f_y(d - d')}
\]
\[
A_s = \frac{K'f_{cu}bd^2}{0.87f_yz} + A'_s - \frac{N}{0.87f_y}
\]

If \( d'/x > 0.43x \),
\[
A'_s = \frac{(K - K')f_{cu}bd^2}{f'_y(d - d')}
\]
\[
A_s = \frac{K'f_{cu}bd^2}{0.87f_yz} + A'_s - \frac{N}{0.87f_y}
\]
\[
f'_y = \left( \frac{x - d'}{0.57x} \right) f_y \quad \text{because steel strain } \varepsilon'_s = \left( \frac{x - d'}{0.57x} \right) \varepsilon_y
\]

where \( \varepsilon_y \) corresponds to steel stress \( f_y/\gamma_m \), as in Section 1.4.2.
Note: The flanged beam becomes a rectangular beam if the bending moment produces tension in the flange.

Design charts in BS 8110: Part 3: 1985\(^{11}\) may be used if design parameters fall within the scope of the charts.

**Step 11  Design for moment — flanged beam**

![Diagram of flanged beam](image)

A flanged beam should be designed as a rectangular beam with width equal to the effective width of flange in compression if \(x \geq 1.1h_t\).

If \(x > 1.1h_t\), find \(b/b_w\) and \(d/h_t\).

Obtain \(\beta_t\) from Fig. 2.1.

Calculate \(M_t = \beta_t f_{ctw} bd^2\)

If \(M_t \geq M_d\),

\[
A_s = \frac{M_d + 0.1f_{ctw} b_w d (0.45d - h_t)}{0.87f_y (d - 0.5h_t)} = \frac{N}{0.87f_y}
\]

If \(M_t < M_d\), follow Section 1.5.4 using the second approach to design of flanged beams.

**Step 12  Maximum allowable shear stress**

Find maximum shear in beam from shear envelope at the face of support or under a concentrated load.

Find \(\nu = V/bd\).

Check \(\nu \leq 0.8 \sqrt{\sigma_{cu}} \leq 5\) N/mm\(^2\)

Change beam geometry if this condition is not satisfied.
Step 13  Design for shear

Select critical sections on beam which are at a distance of 2d from the face of support or concentrated load. Find the following parameters for rectangular and flanged beams.

\[ V = \text{design ultimate shear force} \]

\[ \nu = \frac{V}{bd} \leq 0.8 \sqrt{f_{cu}} \leq 5 \text{N/mm}^2 \]

\[ p = \frac{100A_x}{bd} \]

Find \( \nu_c \) from Figs 11.2 to 11.5.

When axial load in compression, \( N \), is present,

\[ \nu'_c = \nu_c + 0.6 \left( \frac{NVh}{A_cM} \right) \leq 0.8 \sqrt{f_{cu}} \leq 5 \text{N/mm}^2 \]

**Note:** \( N/A_c \) is average stress in the concrete section. \( Vh/M \leq 1 \) and moment and shear at the section under consideration must be for the same load combination. \( N \) is +ve for compression and -ve for tension. To avoid shear cracks at ultimate load, limit shear stress to

\[ \nu'_c = \nu_c \sqrt{1 + \left( \frac{N}{A_c\nu_c} \right)} \]

Replace \( \nu_c \) by \( \nu'_c \) where axial load is present.

Find \( A_{sv} = \frac{0.4bS_x}{0.87f_{yy}} \)

\[ f_{yy} \leq 460 \text{N/mm}^2 \] for links.

Provide minimum area of links, \( A_{sv} \), at a spacing of \( S_y \) for the zone where shear is less or equal to \( V_{nom} \). From the shear force envelope determine zones where \( V \) exceeds \( V_{nom} = (\nu_c + 0.4)bd \)
Find \( A_{sv} = \frac{bS_c(v - v_c)}{0.87f_{sv}} \).

Replace \( v_c \) by \( v'_c \) where axial load is present.

Provide area of links, \( A_{sv} \), at a spacing of \( S \), at the section of the beam in consideration.

For a mixture of links and bent-up bars,

\[ V_b + V_s \geq (v - v_c)bd \]

where

\[ V_b = A_{sb}(0.87f_{sv})(\cos \alpha + \sin \alpha \cot \beta)\frac{(d - d')}{S_b} \quad \alpha \text{ and } \beta \geq 45^\circ \]

\[ V_s = A_{sv}(0.87f_{sv})\frac{d}{S_c} \geq 0.5(v - v_c)bd \]

Replace \( v_c \) by \( v'_c \) where axial load in compression is present.

**Note:** Step 14 below may be omitted if at Step 13 the critical section is selected at a distance of \( d \) from the face of support or from the concentrated load. No further checks will be necessary at the face of support or at the concentrated load.

**Step 14 Alternative design for shear**

Find \( V_{max} = 0.8 \sqrt{f_{cu}} \ 6d \), or \( 5bd \), whichever is less.

Complete the table below:

<table>
<thead>
<tr>
<th>Distance from face of support or concentrated load</th>
<th>( V_{mom} = V_{conc} + 0.4bd )</th>
<th>( V_{conc} &lt; V_{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00d</td>
<td>((v_c + 0.4)bd)</td>
<td>( v_cbd )</td>
</tr>
<tr>
<td>1.75d</td>
<td>((1.143v_c + 0.4)bd)</td>
<td>( 1.143v_cbd )</td>
</tr>
<tr>
<td>1.50d</td>
<td>((1.333v_c + 0.4)bd)</td>
<td>( 1.333v_cbd )</td>
</tr>
<tr>
<td>1.25d</td>
<td>((1.6v_c + 0.4)bd)</td>
<td>( 1.6v_cbd )</td>
</tr>
<tr>
<td>1.00d</td>
<td>((2.0v_c + 0.4)bd)</td>
<td>( 2.0v_cbd )</td>
</tr>
<tr>
<td>0.75d</td>
<td>((2.67v_c + 0.4)bd)</td>
<td>( 2.67v_cbd )</td>
</tr>
<tr>
<td>0.50d</td>
<td>((4v_c + 0.4)bd)</td>
<td>( 4.0v_cbd )</td>
</tr>
<tr>
<td>0.25d</td>
<td>((8v_c + 0.4)bd)</td>
<td>( 8.0v_cbd )</td>
</tr>
</tbody>
</table>
Distance from face of support or concentrated load to

\[
\begin{array}{ccc}
V & V - V_{\text{conc}} \\
\text{(actual shear force)} & \\
2.00d & \\
1.75d & \\
1.50d & \\
1.25d & \\
1.00d & \\
0.75d & \\
0.50d & \\
0.25d & \\
\end{array}
\]

Satisfy the following conditions:
when \( V \leq V_{\text{nom}} \),
\[
A_s = \frac{0.4bS_y}{0.87f_{yw}}
\]
when \( V > V_{\text{nom}} \), calculate
\[
V_s = A_s \left( 0.87f_{yw} \right) \frac{d}{S_y} \geq V - V_{\text{conc}}
\]
The shear resistance may be provided by a combination of links and bent-up bars.

**Step 15 Minimum tension reinforcement**

![Effective width](image)

**SK 2/27** Flanged beam.

**SK 2/28** Rectangular beam.

For flanged beam web in tension, find \( b_w/b \).
If \( b_w/b < 0.4 \)
\[
A_s \geq 0.0018b_w h \quad \text{for} \quad f_y = 460 \text{ N/mm}^2
\]
If \( b_w/b \geq 0.4 \)
\[
A_s \geq 0.0013b_w h \quad \text{for} \quad f_y = 460 \text{ N/mm}^2
\]
For flanged beam flange in tension,
for T-beam  \( A_s \geq 0.0026b_w h \)  for \( f_y = 460 \text{N/mm}^2 \)
for L-beam  \( A_s \geq 0.0020b_w h \)  for \( f_y = 460 \text{N/mm}^2 \)
For rectangular beams,
\( A_s \geq 0.0013bh \)  for \( f_y = 460 \text{N/mm}^2 \)

**Step 16**  **Minimum compression reinforcement — when designed as doubly reinforced**
For flanged beam flange in compression,
\( A'_s \geq 0.004bh_t \)
For flanged beam web in compression,
\( A'_s \geq 0.002b_w h \)
For rectangular beam,
\( A'_s \geq 0.002bh \)

*Note:* Minimum compression reinforcement in beams will be used only when compression reinforcement is required.

**Step 17**  **Minimum transverse reinforcement in flange**

![Diagram of flanged beam](SK 2/29 Minimum transverse reinforcement in flange of flanged beam.)

For flanged beams over full effective flange width near top surface, use \( 1.5h_t \text{mm}^2/\text{m} \) reinforcement for the whole length of the beam. Normally this amount of reinforcement is provided in the slab at the top surface over the beam as part of slab reinforcement when the flanged beam forms part of a beam—slab construction.

**Step 18**  **Minimum reinforcement in side face of beams**

\[ d = \text{dia. of bar} \geq \sqrt{\left( \frac{S_b h}{f_y} \right)} \]

\( S_b \leq 250 \text{ mm} \)
\( b = \text{actual, or 500 mm, whichever is the lesser.} \)
SK 2/30 Minimum reinforcement side face of beam.

Note: To control cracking on the side faces of beams use small diameter bars at close spacings. The distribution of these bars should be over two-thirds of beam's overall depth measured from tension face.

\[ A_s \geq 0.00125bh \] on each side face as shown.

**Step 19 Deflection**

SK 2/31 Doubly reinforced flanged beam.

SK 2/32 Simply supported or continuous beam. \( M = \) moment at midspan.

SK 2/33 Cantilever beam. \( M = \) moment at support.

Find \( b_w/b \) for flanged beams.
Find \( l_e/d \).
Find basic span/effective depth ratio from Table 11.3.

Note: If \( b_w/b \) is greater than 0.3, then interpolate between values in Table 11.3 assuming \( b_w/b \) equal to 1 for rectangular beams and 0.3 for flanged beams.
Find service stress \( f_s = f_y \left( \frac{5}{8\beta_0} \right) \left( \frac{A_{s\text{ reqd}}}{A_{s\text{ prov}}} \right) \)

where \( \beta_0 = M/M' \)
\( M = \) moment after redistribution
\( M' = \) moment before redistribution
\( A_{s\text{ reqd}} = \) area of steel required from calculations
\( A_{s\text{ prov}} = \) area of steel actually provided.

Find \( M/\beta d^2 \).

Find modification factor for tension reinforcement from Chart 11.5.

Find \( 100A_{s}/bd \).

Find modification factor for compression reinforcement from Chart 11.4.

Find modified span/depth ratio by multiplying the basic span/depth ratio by the modification factor of tension and compression reinforcement.

Check \( l/\beta d < \) modified span/depth ratio

**Note:** Table 11.3 can be used for up to a 10 m span. Beyond a 10 m span multiply these values by 10/span except for cantilevers where deflection should be calculated (see Section 1.8 for calculation of deflection.)

**Step 20 Maximum areas of reinforcement**

For all beams,
\( A_s \leq 0.04b_d h \)
\( A_{s'} \leq 0.04b_w h \)

**Step 21 Containment of compression reinforcement**

Designed compression reinforcement in a beam should be contained by links.
Minimum diameter of links = 0.25 times diameter of largest compression bar, or 6 mm, whichever is greater.
Maximum spacing of links = 12 times diameter of smallest bar in compression.

'd' is greater of 0.25D or 6mm

SK 2/34 Containment of compression reinforcement.
Step 22 Bearing stress inside bend

Check bearing stress inside bend where it is required to extend the bar for more than 4 x diameter beyond the bend because the anchorage requirement is not otherwise satisfied. Satisfy that

$$\text{bearing stress} = \frac{F_{bt}}{r \phi} \leq \frac{2f_{cu}}{1 + 2\left(\frac{\phi}{a_0}\right)}$$

where \(F_{bt}\) = tension in bar at the start of bend
\(a_0\) = centre to centre of bar, or, cover plus diameter of bar.

Step 23 Curtailment of bars

Follow simplified detailing rules for beams where the load is predominantly uniformly distributed and spans in a continuous beam are approximately equal. Follow bending moment diagram for other cases.

Step 24 Spacing of bars

Minimum clear spacing horizontally = \(MSA + 5 \geq \text{diameter of bar}\)

where \(MSA\) = maximum size of aggregate.

Minimum clear spacing vertically between layers = \(\frac{2MSA}{3}\)

Maximum clear spacing of bars in tension \(\leq \frac{47000}{f_s} \leq 300\)

Service stress in bar \(f_s = f_y \left(\frac{5}{8b_0}\right) \left(\frac{A_{s \text{ regl}}}{A_{s \text{ prov}}}ight)\)
(See Step 19 for explanation of $\beta_h$.)

The distance between the corner of the beam and the nearest longitudinal bar in tension should not be greater than half the maximum clear spacing.

**Note:** In normal internal or external condition of exposure where the limitation of crack widths to 0.3 mm is appropriate, Step 24 will deem to satisfy the crack width criteria.

**Step 25  Torsional shear stress**

Check torsional shear stress.
Find ultimate torsion $T$ from analysis.
For a rectangular section, torsional shear stress, $\nu_t$, is given by

$$\nu_t = \frac{2T}{h_{\text{min}}^2\left(h_{\text{max}} - \frac{h_{\text{min}}}{3}\right)}$$

For I, T or L-section, divide each section into component rectangles.

Proportion of total torsion carried by each rectangle $= T_s$

$$\frac{Th_{\text{min}}^2 h_{\text{max}}}{\sum (h_{\text{min}}^3 h_{\text{max}})} = T_s$$

Torsional shear stress for each section

$$\nu_t = \frac{2T_s}{h_{\text{min}}^2\left(h_{\text{max}} - \frac{h_{\text{min}}}{3}\right)}$$
For hollow and other box sections, follow the method in Chapter 8. If wall thickness in a rectangular hollow section exceeds one quarter of the dimension in that direction, treat the hollow section as a solid rectangle. Calculate

\[ v_{t,\text{min}} = 0.067 \sqrt{f_{\text{cu}}} < 0.4 \text{ N/mm}^2 \]
\[ v_{\text{tu}} = 0.8 \sqrt{f_{\text{cu}}} \leq 5 \text{ N/mm}^2 \]

If \( v_t < v_{t,\text{min}} \), no torsional reinforcement is required.

**SK 2/38** Torsional reinforcement in beam.

If \( v_{t,\text{min}} < v_t < v_{\text{tu}} \), provide torsional shear reinforcement by closed links and longitudinal bars.

Check \( (v + v_t) < v_{\text{tu}} \)

where \( v \) = flexural shear stress.

Check \( v_t < v_{\text{tu}} \left( \frac{y_1}{550} \right) \)

\[ A_{sv} \geq \frac{T}{S_y \cdot 0.8 x_1 y_1 (0.87 f_y)} \]
\[ A_s \geq \frac{A_{sv} f_y (x_1 + y_1)}{S_y f_y} \]

**Note:** Add torsional reinforcement to already calculated shear reinforcement.

\( S_v < x_1 \), or \( \frac{y_1}{2} \), or 200 mm, whichever is the least

**Step 26** Crack width in flexure

**Serviceability limit state**

Load combination \( LC_7 = 1.0DL + 1.0LL + 1.0EP + 1.0WP + 1.0WL \)

**Note:** Omit loadings from \( LC_7 \) which produce beneficial rather than adverse effect.
SK 2/40 Critical dimensions for crack width calculations.

\[
\frac{b_t (h-x)(d'-x)}{3E_s A_s (d-x)}
\]

SK 2/41 Strain diagram for crack width calculations.

SK 2/42 Doubly reinforced rectangular beam.

\[
W_{\text{max}} = \frac{3a_{cr} \varepsilon_m}{1 + 2(a_{cr} - \varepsilon_{\text{min}})} \quad \frac{2(h - x)}{(h - x)}
\]

\[
\varepsilon_m = \varepsilon_1 - b_t (h - x) \quad \frac{(a' - x)}{3E_s A_s (d - x)}
\]

Note: \(\varepsilon_1\) is the strain due to load combination \(LC_7\).
where \( b_i \) is the width of the section at the centroid of tensile reinforcement.

For a rectangular section,

**Note:** A flanged beam is a rectangular section if \( x \leq 1.1h_i \).

\[
m = \frac{E_s}{E_c}
\]

\[
x = d' \left( \frac{(mp + (m - 1)p')^2 + 2(mp + (m - 1)\left(\frac{d'}{d}\right)p')^{\frac{1}{2}}}{- (mp + (m - 1)p')} \right)
\]

(See Section 1.13.1.)

\[
p = \frac{A_s}{bd} \quad p' = \frac{A_s'}{bd}
\]

\[
f_c = \frac{M}{k_2bd^2 + k_3A_s(d - d')}
\]

\[
k_2 = \left( \frac{x}{2d} \right) \left( 1 - \frac{x}{3d} \right)
\]

\[
k_3 = (m - 1) \left( 1 - \frac{d'}{x} \right)
\]

\[
f_s = mf_c \left( \frac{d}{x} - 1 \right)
\]

\[
\varepsilon_s = \frac{f_s}{E_s}
\]

\[
\varepsilon_h = \left( \frac{h - x}{d - x} \right) \varepsilon_s
\]

\[
\varepsilon_{mh} = \varepsilon_h - \frac{b(h - x)^2}{3 E_c A_s(d - x)}
\]

**Note:** In normal internal or external condition of exposure where the limitation of crack widths to 0.3 mm is appropriate, Step 24 will deem to satisfy the crack width criteria.

**Step 27** Design of connections to other components

Follow Chapter 10.

### 2.4 WORKED EXAMPLES

**Example 2.1** Simply supported rectangular beam

Clear span = 6.0 m
Overall depth = 500 mm
Width = 300 mm
Width of supporting walls = 200 mm
All reinforcement to be used is high yield steel with $f_y = 460 \text{N/mm}^2$.

Note: Steps 1–5 form part of the analysis and are excluded from the worked example. For a typical analysis see Example 2.3.

Step 6 Determination of cover
- Maximum size of aggregate = 20 mm
- Maximum bar size assumed = 32 mm
- Maximum size of link assumed = 10 mm
- Exposure condition = severe
- Fire resistance required = 2 hours.

Refer to the following tables in Chapter 11:

Table 11.6 grade of concrete = C40 for severe exposure
Table 11.6 minimum cement content = 325 kg/m$^3$
Table 11.6 maximum free water/cement ratio = 0.55
Table 11.6 nominal cover = 40 mm
Table 11.7 nominal cover to beams for 2 hours fire resistance = 40 mm

For 2 hours fire resistance, minimum width of beam = 200 mm, from Figure 3.2 of BS8110: Part 1: 1985.\[1\]
Effective depth, $d$, is given by:

\[ d = \text{overall depth} - \text{nominal cover} - \text{dia. of link} - \text{half dia. of bar} = 500 - 40 - 10 - 16 = 434 \text{mm} \]

**Step 7** Determination of effective span

\[ l + d = 6.0 + 0.434 = 6.434 \text{m} \]

\[ l_o = 6.2 \text{m} \]

Therefore \[ l_c = l_o = 6.2 \text{m} \]

**Step 8** Determination of effective width

Not required.

**Step 9** Check slenderness of beam

\[ l = 6.0 \text{m} \]

\[ 60b_c = 60 \times 300 \text{mm} = 18.0 \text{m} \]

\[ \frac{250 b_c^2}{d} = \frac{250 \times 300^2}{434} = 51.84 \text{m} \]

Satisfied \[ l < 60b_c < \frac{250b_c^2}{d} \]

**Step 10** Design for moment – rectangular beam

Maximum ultimate bending moment = 216 kNm

Maximum shear at face of support = 140 kN

Shear at 2d from face of support = 96 kN

Shear at d from face of support = 116 kN

Direct load, \( N = 0 \text{kN} \)

\( M_d = M = 216 \text{kNm} \)

\( f_{cu} = 40 \text{N/mm}^2 \)
\[ K = \frac{M_d}{f_{cd}bd^2} = \frac{216 \times 10^6}{40 \times 300 \times 434^2} = 0.0956 < 0.156 \]

\[ z = d \left[ 0.5 + \sqrt{\left( 0.25 - \frac{K}{0.9} \right)} \right] = d \left[ 0.5 + \sqrt{\left( 0.25 - \frac{0.0956}{0.9} \right)} \right] = 0.88d = 382 \text{ mm} \]

\[ x = \frac{d - z}{0.45} = \frac{434 - 382}{0.45} = 116 \text{ mm} \]

\[ A_s = \frac{M_d}{0.87f_yz} = \frac{216 \times 10^6}{0.87 \times 460 \times 382} = 1413 \text{ mm}^2 \]

Use 3 no. 25 mm dia. Grade 460 = 1472 mm²

**Step 11** Design for moment – flanged beam

Not required.

**Step 12** Check maximum allowable shear

\[ v = \frac{V}{bd} \text{ at face of support} \]

\[ = \frac{140 \times 10^3}{300 \times 434} = 1.075 \text{ N/mm}^2 < 0.8 \sqrt{f_{cu}} = 5 \text{ N/mm}^2 \]

**Step 13** Design for shear

2\(d = 870 \text{ mm} \)

\[ V = 96 \text{ kN} \text{ at } 2d \text{ from support face} \]

\[ v = \frac{96 \times 10^3}{300 \times 434} = 0.74 \text{ N/mm}^2 < 0.8 \sqrt{f_{cu}} = 5 \text{ N/mm}^2 \]

\[ p = \frac{100 A_s}{bd} = \frac{100 \times 1472}{300 \times 434} = 1.13\% \]

\[ v_c = 0.65 \times 1.17 \text{ N/mm}^2 \text{ for Grade 40 concrete} \]

\[ = 0.76 \text{ N/mm}^2 \text{ from Fig. 11.5} \]
Design of Reinforced Concrete Beams

\[ V_{\text{nom}} = (\nu_c + 0.4) bd \]
\[ = 151 \text{kN} > 140 \text{kN} \quad \text{at face of support} \]
\[ V < V_{\text{nom}} \text{ at all points in the beam.} \]

Nominal links \( A_{sv} = \frac{0.4b_{sv}}{0.87f_{sv}} \)

Assume \( S_v = 300 \text{mm} \)

\[ A_{sv} = \frac{0.4 \times 300 \times 300}{0.87 \times 460} \]
\[ = 90 \text{mm}^2 \]

Use 8 mm dia. single closed link = \( A_{sv} = 100 \text{mm}^2 \) \((f_y = 460 \text{N/mm}^2)\) at 300 mm centre to centre.

**Step 14 Alternative design for shear**

\( V_{\text{nom}} > V \) at face of support so Step 14 is superfluous – use nominal links everywhere on the beam.

**Step 15 Minimum tensile reinforcement**

Minimum tensile reinforcement = \( 0.0013bh \)

\[ = 0.0013 \times 300 \times 500 \text{mm}^2 \]
\[ = 195 \text{mm}^2 < 1472 \text{mm}^2 \text{ provided} \]

2 no. 12 diameter (= 226 mm\(^2\)) provided at top of beam.

**Step 16 Minimum compression reinforcement**

Not required.

**Step 17 Minimum transverse reinforcement in flange**

Not required.

**Step 18 Minimum reinforcement in side face of beams**

---

SK 2/46 Section through beam.
\[ b = \text{actual, or } 500\text{mm, whichever is the lesser.} \]

Minimum dia. of bar in side face of beam = \( \sqrt{\frac{S_b b}{f_y}} \)  

(assume \( S_b = 200\text{mm} \))

\[ = \sqrt{\frac{200 \times 300}{460}} = 11.4\text{mm} \]

Use 12 dia. Grade 460 bars at approximately 200 centres on the side face of beam.

Reinforcement on each side face of beam = 2 no. 12 dia. + 1 no. 25 dia.  
= 716 mm\(^2\)

\[ A_s = 0.00125bh \]
\[ = 0.00125 \times 300 \times 500 \]
\[ = 188\text{mm}^2 < 716\text{mm}^2 \quad \text{OK} \]

**Note:** Strictly speaking these bars on the side face are not required for beams less than 750mm overall depth but it is good practice to use them in order to avoid shrinkage cracks.

**Step 19** Check deflection

\[ \frac{l_c}{d} = \frac{6200}{434} = 14.3 \]

Basic span/depth ratio = 20  
from Table 11.3

\[ \beta_b = 1, \]

\[ f_s = f_y \left( \frac{S}{8} \right) \left( \frac{A_{s \text{ reqd}}}{A_{s \text{ prov}}} \right) = 460 \left( \frac{5}{8} \right) \left( \frac{1413}{1472} \right) = 275 \text{N/mm}^2 \]

\[ \frac{M}{bd^2} = \frac{216 \times 10^6}{300 \times 434^2} = 3.8 \]

Modification factor for tension reinforcement = 0.90  
from Table 11.5

Modified span/depth ratio = \( 20 \times 0.90 = 18 > \frac{l_c}{d} = 14.3 \)

Hence deflection is OK.

**Step 20** Maximum areas of reinforcement

\( A_s \) is less than 4%.

**Step 21** Containment of compression reinforcement

Not required.

**Step 22** Check bearing stress inside bend

Not required.
Step 23  Curtailment of bars

\[ 0.08l = 0.08 \times 6000 = 480 \text{ mm} \]

The central 25 mm dia. bar will be stopped 250 mm from the face of the support.

Step 24  Spacing of bars

Clear spacing between bars in tension = 64.5 mm
Minimum required spacing = 20 + 5 = 25 mm

Maximum spacing = \[ \frac{47000}{f_s} \]
= \[ \frac{47000}{275} = 170 \text{ mm} \]

where \( f_s = 275 \text{ N/mm}^2 \)  (see Step 19)
Spacing of bars is OK.

Step 25  Check torsional shear stress

Not required.

Step 26  Crack width calculations

Service maximum moment = 144 kNm
\( A_s = 1472 \text{ mm}^2 \quad d'' = 54 \text{ mm} \)
\( A'_s = 226 \text{ mm}^2 \quad d = 439 \text{ mm} \)

\( m = \frac{E_s}{E_c} \)
\( m = \frac{200}{20} = 10 \)

\( E_c \) assumed halfway between long and short-term.
SK 2/49 Dually reinforced rectangular beam.

SK 2/50 Strain diagram.

SK 2/51 Crack width calculations.

\[ p = \frac{A_s}{bd} = \frac{1472}{300 \times 439} = 0.0112 \quad p' = \frac{A'_s}{bd} = 0.0017 \]

\[ x = d \left[ (mp + (m - 1)p')^2 + 2(mp + (m - 1)\left(\frac{d'}{d}\right)p') \right]^{1} \]

\[ (mp + (m - 1)p') = 160 \text{ mm} \]

\[ k_2 = \left( \frac{x}{2d} \right) \left( 1 - \frac{x}{3d} \right) \]

\[ = \left( \frac{160}{2 \times 439} \right) \left( 1 - \frac{160}{3 \times 439} \right) \]

\[ = 0.16 \]

\[ k_3 = (m - 1) \left( 1 - \frac{d'}{x} \right) = (10 - 1) \left( 1 - \frac{54}{160} \right) = 5.96 \]

\[ f_c = \frac{M}{k_2bd^2 + k_3A_s(d - d')} \]

\[ = \frac{144 \times 10^6}{0.16 \times 300 \times 439^2 + 5.96 \times 226 \times (439 - 54)} \]

\[ = 14.74 \text{ N/mm}^2 \]

\[ f_s = mf_c \left( \frac{d}{x} - 1 \right) \]
Design of Reinforced Concrete Beams

\[= 10 \times 14.74 \times \left(\frac{439}{160} - 1\right)\]

\[= 257 \text{ N/mm}^2\]

\[\varepsilon_s = \frac{f_s}{E_s} = \frac{257}{200 \times 10^3} = 1.285 \times 10^{-3}\]

\[\varepsilon_h = \left(\frac{h - x}{d - x}\right)\varepsilon_s = \left(\frac{340}{279}\right)\varepsilon_s = 1.566 \times 10^{-3}\]

\[\varepsilon_{mh} = \varepsilon_h - \frac{b(h - x)^2}{3E_sA_s(d - x)}\]

\[= 1.566 \times 10^{-3} - \frac{300 \times 340^2}{3 \times 200 \times 10^3 \times 1472 \times 279}\]

\[= 1.425 \times 10^{-3}\]

\[a_{c1} = 1.414 \times 60.5 - 12.5 = 73.0 \text{ mm}\]

\[a_{c2} = \sqrt{(60.5^2 + 45^2)} - 12.5 = 62.9 \text{ mm}\]

\[a_{c1} > a_{c2}\]

\[W_{cr} = \frac{3a_{c1} \varepsilon_m}{1 + \left[\frac{2(a_{c1} - C_{min})}{h - x}\right]}\]

\[= \frac{3 \times 73.0 \times 1.425 \times 10^{-3}}{1 + \left[\frac{2(73 - 48)}{340}\right]}\]

\[= 0.27 \text{ mm} < 0.3 \text{ mm} \quad \text{OK}\]

Example 2.2 Three span continuous beam

SK 2/52 Three-span continuous beam.

Three equal spans of 10 m centre-to-centre of columns.

Width of column = 0.4 m
clear span = 9.6 m
slab depth = 150 mm
beam spacing = 4.0 m
beam overall depth = 550 mm
beam width = 300 mm

Redistribution of moments = 10%

Note: Steps 1–5 form part of the analysis and have been excluded. For a typical analysis see Example 2.3.
All reinforcement to be used will be high yield steel with $f_y = 460 \text{ N/mm}^2$. It is expected that the analysis will be carried out using a computer program with the load combination shown in Section 2.2.

From moment and shear envelope,

- $M_A = 0$  $V_{AB} = 300 \text{ kN}$  $V_{A'B} = 250 \text{ kN}$
- $M_{AB} = +600 \text{ kNm}$  $V_{AB} = $ negligible
- $M_B = -650 \text{ kNm}$  $V_{BA} = 370 \text{ kN}$  $V_{B'A} = 320 \text{ kN}$
- $V_{BC} = 325 \text{ kN}$  $V_{B'C} = 275 \text{ kN}$
- $M_{BC} = +370 \text{ kNm}$ or $-150 \text{ kNm}$

where $V_{A'B}$ = shear at a distance of $d$ from face of support.

**Step 6 Determination of cover**

- Maximum size of aggregate = 20 mm
- Maximum bar size = 32 mm
- Maximum size of link = 8 mm
- Exposure condition = Severe
- Fire resistance required = 2 hours
- Grade of concrete = C40
- Maximum cement content = 325 kg/m$^3$
- Maximum free water cement ratio = 0.55
- Nominal cover = 40 mm from Tables 11.6 and 11.7
- Effective depth, $d = 550 - 40 - 8 - 16 = 486 \text{ mm}$

**Step 7 Effective span**

$L_e = l_o = 10.0 \text{ m}$

**Step 8 Effective width of compression flange**

Actual $b = 4.0 \text{ m}$ (centre-to-centre of beams)

Calculated $b = \frac{L_e}{7.14} + b_w$

$= \frac{10000}{7.14} + 300$

$= 1700 \text{ mm}$

![Diagram](image)

**SK 2/53 Effective width of compression flange.**
Step 9  Slenderness check
May be ignored.

Step 10  Design for moment
\( M_{AB} = 600 \text{kNm} \)

**Flanged beam**
\( M_d = M_{AB} = 600 \text{kNm} \)

\[
K = \frac{M_d}{f_{cb}bd^2}
= \frac{600 \times 10^6}{40 \times 1700 \times 486^2}
= 0.0373
\]

\[
z = d \left[ 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right]
= d \left[ 0.5 + \sqrt{0.25 - \frac{0.0373}{0.9}} \right]
= 0.95d = 462 \text{mm}
\]

\[
x = \frac{d - z}{0.45}
= \frac{486 - 462}{0.45}
= 53 \text{mm} < h_t = 150 \text{mm}
\]

Neutral axis in the slab

\[
A_k = \frac{M_d}{0.87fyz}
= \frac{600 \times 10^6}{0.87 \times 460 \times 462} = 3245 \text{mm}^2
\]

Use 3 no. 32 dia. bars in bottom layer plus 2 no. 25 dia. bars in top layer.

---

**SK 2/54** Arrangement of reinforcement at bottom of beam at midspan.
Total area of steel provided = 3394 mm²

Check effective depth.
Centre of gravity of group of 5 bars

\[ x = \frac{3 \times 804 \times 64 + 2 \times 491 \times 124.5}{3394} \]

= 81.5 mm

\[ d = 550 - 81.5 = 468.5 \text{ mm} \]

Recheck reinforcement requirement with revised effective depth:

\[ K = 0.040 \]

\[ z = 0.95 \times 468.5 = 445 \text{ mm} \]

\[ A_s = 3369 \text{ mm}^2 \quad \text{(required)} \]

\[ A_s \text{ provided} = 3394 \text{ mm}^2 \quad \text{OK} \]

\[ M_B = -650 \text{ kNm} \]

**Rectangular beam**

\[ M_d = 650 \text{ kNm} \]

Effective depth, \( d = 550 - 40 - 32 - 16 - 8 = 454 \text{ mm} \)

(assuming two layers of 32 dia. bars)

\[ K = \frac{650 \times 10^6}{40 \times 300 \times 454^2} \]

= 0.263 > 0.156

Compression reinforcement required.

Redistribution is 10%

\[ A'_s = \frac{(K - 0.156)f_{cb}bd^2}{0.87f_y(d - d')} \]

\[ = \frac{(0.263 - 0.156) \times 40 \times 300 \times 454^2}{0.87 \times 460 \times (454 - 64)} \]

= 1696 mm²

Use 3 no. 32 dia. bars (2412 mm²) – bottom of beam.

\[ z = d \left[ 0.5 \sqrt{\left( 0.25 - \frac{0.156}{0.9} \right)} \right] \]

= 0.775d = 352 mm

\[ x = 0.5d = 227 \text{ mm and } \frac{d'}{x} = \frac{64}{227} = 0.28 < 0.43 \]

\[ A_s = \frac{0.156f_{cb}bd^2}{0.87f_yx} + A'_s \]
Design of Reinforced Concrete Beams

\[ \frac{0.156 \times 40 \times 300 \times 454^2}{0.87 \times 460 \times 352} + 1696 = 4435 \text{mm}^2 \]

Use 6 no. 32 dia. bars (4824 mm\(^2\)) – top of beam in two layers.

![Diagram showing beam arrangement](image)

SK 2/55 Arrangement of reinforcement at top of beam over support.

\( M_{BC} = +370 \text{ kNm} \)

**Flanged beam**

\( b = 1700 \text{ mm} \)

\( d = 550 - 40 - 8 - 16 = 486 \text{ mm} \)

\( K = \frac{M_d}{f_{cu}bd^2} = 0.023 \)

\( z = 0.95d = 426 \text{ mm} \)

\( A_k = \frac{M_d}{0.87f_yz} = \frac{370 \times 10^6}{0.87 \times 460 \times 462} = 2001 \text{ mm}^2 \)

Use 3 no. 32 dia. bar (2412 mm\(^2\)) – bottom of beam

\( M_{BC} = -150 \text{ kNm} \)

**Rectangular beam**

\( b = 300 \text{ mm} \)

\( d = 486 \text{ mm} \)

\( K = \frac{150 \times 10^6}{300 \times 486^2 \times 40} = 0.053 \)


\[ z = d \left[ 0.5 + \sqrt{\left( 0.25 - \frac{K}{0.9} \right)} \right] \]

\[ = 0.94d = 456 \text{ mm} \]

\[ A_s = \frac{150 \times 10^6}{0.87 \times 460 \times 456} = 822 \text{ mm}^2 \]

Use 2 no. 32 dia. bar (1608 mm\(^2\)) — top of beam.

**Step 11** Design for moment — flanged beam

Not required.

**Step 12** Maximum shear stress

\[ \nu = \frac{V_{\text{max}}}{bd} = 2.716 \text{ N/mm}^2 < 5 \text{ N/mm}^2 \]

**Step 13** Design for shear

Maximum shear = 370 kN = \(V_{BA}\)

\[ \nu = \frac{V}{bd} = \frac{370 \times 10^3}{300 \times 454} \]

\[ = 2.716 \text{ N/mm}^2 < 0.8 \sqrt{f_{cu}} = 5 \text{ N/mm}^2 \quad \text{OK} \]

Check shear stress at \(d\) from face of column.

\(V_{AB} = 250 \text{ kN}\)

\(d = 468.5 \text{ mm} \quad \text{for span AB}\)

\[ \nu = \frac{250 \times 10^3}{300 \times 468.5} \]

\[ = 1.78 \text{ N/mm}^2 \]

\[ \frac{p}{bd} = \frac{100A_s}{300 \times 468.5} \]

\[ = 2.41 \]
\[ v_c = 0.85 \times 1.17 \quad \text{from Fig. 11.5} \]
\[ = 0.99 \text{ N/mm}^2 \]

\[ V_{\text{nom}} = (v_c + 0.4)bd \]
\[ = 195 \text{ kN} \]

\[ v > v_c + 0.4 = 1.39 \text{ N/mm}^2 \]

\[ A_{sv} = \frac{bS_v(v - v_c)}{0.87f_y} \]

(assume \( S_v = 150 \text{ mm} \))
\[ = \frac{300 \times 150 \times (1.78 - 0.99)}{0.87 \times 460} \]
\[ = 89 \text{ mm}^2 \]

Use 8 mm dia. links = 100 mm\(^2\) (two legs) at 150 centre-to-centre up to the point where shear falls to 195 kN. High yield reinforcement (\( f_y = 460 \text{ N/mm}^2 \)).

Nominal \( A_{sv} = \frac{0.4bS_v}{0.87f_y} \)
\[ = \frac{0.4 \times 300 \times 300}{0.87 \times 460} = 90 \text{ mm}^2 \]

Use 8 mm dia. links = 100 mm\(^2\) (two legs) at 300 centre-to-centre as nominal links (\( f_y = 460 \text{ N/mm}^2 \)).

\[ V_{B'A} = 320 \text{ kN} \]

\[ v = \frac{320 \times 10^3}{300 \times 454} \quad (d = 454 \text{ mm at B}) \]
\[ = 2.35 \text{ N/mm}^2 \]

\[ p = \frac{100A_s}{bd} = \frac{100 \times 4435}{300 \times 454} = 3.25 \]

\[ v_c = 0.91 \times 1.17 = 1.065 \text{ N/mm}^2 \quad \text{from Fig. 11.5} \]

\[ A_{sv} = \frac{bS_v(v - v_c)}{0.87f_y} \]
\[ = \frac{300 \times 150 \times (2.35 - 1.063)}{0.87 \times 460} \]
\[ = 144.5 \text{ mm}^2 \]

Use 8 mm dia. links = 150 mm\(^2\) (3 legs) at 150 centre to centre up to the point where shear falls to 195 kN.

**Step 14** Alternative design for shear
Omitted.

**Step 15** Minimum tension reinforcement
**Flanged beam**

\[
\frac{b_w}{b} = \frac{300}{1700} = 0.176 < 0.4
\]

For web in tension

\[A_s > 0.0018b_w h = 297 \text{ mm}^2\]

For flange in tension

\[A_s > 0.0026b_w h = 429 \text{ mm}^2\]

Both conditions satisfied.

**Step 16 Minimum compression reinforcement**

\[A'_c > 0.002b_w h = 330 \text{ mm}^2\]

Provided \(A'_c = 2412 \text{ mm}^2\)

Condition satisfied.

**Step 17 Transverse reinforcement in flange**

Minimum transverse reinforcement in flange = \(1.5h_t \text{ mm}^2/\text{m}\)

\[= 1.5 \times 150 \text{ mm}^2/\text{m}\]

\[= 225 \text{ mm}^2/\text{m}\]

Reinforcement in the slab over the beam will be a lot more than this quantity.

**Step 18 Reinforcement in side face of beam**

For a 550 mm overall depth of beam with 150 mm slab, side reinforcement will not be required.

**Step 19 Check deflection**

\[
\frac{b_w}{d} = \frac{10000}{468.5} = 21.3
\]

\[d = 468.5 \text{ mm} \quad \text{for span AB}\]

\[
\frac{b_w}{b} = 0.176 < 0.3
\]

Basic span/effective depth ratio from Table 11.3 = 20.8

Since the ultimate moment at midspan is greater after redistribution than the ultimate elastic moment, the service elastic stress may be taken as \((5/8)f_y\).

Service stress, \(f_s = \frac{5}{8}f_y\) (assumed)

\[
= \frac{5}{8} \times 460
\]

\[= 288 \text{ N/mm}^2\]
Design of Reinforced Concrete Beams

\[
\frac{A_{s\,\text{reqd}}}{A_{s\,\text{prov}}} = \frac{3369}{3394} = 1.0
\]

\[
\frac{M}{bd^2} = \frac{600 \times 10^6}{1700 \times 468.5^2} = 1.6
\]

Modification factor = 1.19 from Chart 11.5
Modified span/effective depth ratio = 20.8 × 1.19
\[
= 24.75 > 21.3 \quad \text{OK}
\]

**Step 20**  
**Maximum areas of reinforcement**

\[A_k = 0.04b_e h = 6600 \text{ mm}^2\]

Maximum tensile reinforcement used = 4824 mm²  
**OK**

**Step 21**  
**Containment of compression reinforcement**

Minimum dia. of links = 0.25 × max. dia. of bar
\[= 0.25 \times 32 = 8 \text{ mm} \quad \text{OK}\]

Maximum spacing of links = 12 × dia. of bar
\[= 12 \times 32 \text{ mm} = 384 \text{ mm} \quad \text{OK}\]

**Note:** At least one link at the centre of columns B and C will be required for containment.

**Step 22**  
**Check bearing stress inside bend**

Not required.

**Step 23**  
**Curtailment of bars**

\[0.15l = 1500 \text{ mm}\]
\[0.10l = 1000 \text{ mm}\]
\[0.25l = 2500 \text{ mm}\]

**Span AB**

Continue 3 no. 32 dia. + 2 no. 32 dia. up to 1000 mm from A (end support).

Stop 1 no. 32 dia. and 2 no. 32 dia. at 1000 mm from A.

(See Step 26: reinforcement in span AB increased.)

**Over support B (top bars)**

Continue 6 no. 32 dia. bar top up to 1500 mm on either side of B.

Stop 2 no. 32 dia. bar at 1500 from B.

Stop 2 no. 32 dia. bar at 2500 from B.

Continue 2 no. 32 dia. through span.

**Step 24**  
**Spacing of bars**

Minimum clear spacing = \(MSA + 5 = 25\text{ mm}\)
Clear spacing of bars in tension = 54 mm ≥ 25 mm

Maximum clear spacing = \( \frac{47\,000}{f_s} = \frac{47\,000}{288} = 163 \text{ mm} \)

(See Step 19 for \( f_s \))
At span BC top tension reinforcement
Clear spacing of bars (2 no. 32 dia.) = 140 mm OK

**Note:** Under normal circumstances this step will deem to satisfy the 0.3 crack width limitation criteria, but, as Step 26 will prove, when crack width calculations are actually carried out this may not be the case. In span AB the maximum clear spacing criterion is satisfied but the calculations show that the crack widths may be exceeded.

**Step 25** Check torsional shear stress
Not required.

**Step 26** Crack width calculations

*Span AB*
Maximum service moment = 400 kNm

\( d = 468.5 \text{ mm} \quad C_{\text{min}} = 48 \text{ mm} \)

\( b = 1700 \text{ mm} \)

\( A_s = 3394 \text{ mm}^2 \)

\( A_s' = 1608 \text{ mm}^2 \quad \text{(ignored in the computation)} \)

\( m = \frac{E_s}{E_c} = 10 \)

\( x = \sqrt{\left( \frac{(mA_s)^2}{b} \right) + \frac{2mA_s d}{b}} - \frac{mA_s}{b} \)

\( = 118 \text{ mm} < h_t (= 150 \text{ mm}) \)

\( z = d - \frac{x}{3} = 429 \text{ mm} \)

\( f_s = \frac{400 \times 10^6}{429 \times 3394} = 274 \text{ N/mm}^2 \)

\( \varepsilon_s = \frac{f_s}{E_s} = \frac{274}{200 \times 10^3} = 1.37 \times 10^{-3} \)

\( \varepsilon_h = \left( \frac{h - d}{d - x} \right) \varepsilon_s = \left( \frac{550 - 118}{468.5 - 118} \right) \times 1.37 \times 10^{-3} = 1.69 \times 10^{-3} \)
\[ \varepsilon_{mh} = \varepsilon_h - \frac{b(h - x)^2}{3E_cA_s(d - x)} \]

\[ = 1.69 \times 10^{-3} - \frac{300 \times 432^2}{3 \times 200 \times 10^3 \times 350.5 \times 3394} \]

\[ = 1.61 \times 10^{-3} \]

\[ a_{e1} = \sqrt{(64^2 + 64^2)} - 16 = 74.5 \text{ mm} \]

\[ a_{e2} = \sqrt{(64^2 + 43^2)} - 16 = 61.1 \text{ mm} \]

\[ a_{er} = 74.5 \text{ mm} \quad \text{at the corner of the beam} \]

\[ W_{cr} = \frac{3a_{er} \varepsilon_{em}}{1 + \frac{2(a_{er} - c_{min})}{(h - x)}} \times \frac{3 \times 74.5 \times 1.61 \times 10^{-3}}{1 + \frac{2(74.5 - 48)}{(550 - 118)}} \]

\[ = 0.32 \text{ mm} > 0.3 \text{ mm} \]

The calculated crack width is greater than allowable. Increase reinforcement to 5 no. 32 dia. bar instead of 3 no. 32 dia. plus 2 no. 25 dia. No more checks are necessary.

*Over support B*

![Arrangement of bars over support](image1)

SK 2/57 Arrangement of bars over support.

At face of column,

maximum service moment = 390 kNm

\[ d = 454 \text{ mm} \quad d' = 64 \text{ mm} \]

\[ b = 300 \text{ mm} \]

\[ A_s = 4824 \text{ mm}^2 \]

\[ A_4 = 2412 \text{ mm}^2 \]
\[ C_{\text{min}} = 48 \text{ mm} \]

\[ m = 10 \]

See Step 26 of Example 2.1 for explanation of symbols and the equations.

\[ x = 225 \text{ mm} \]

\[ K_2 = 0.2068 \]

\[ K_3 = 6.44 \]

\[ f_c = \frac{M}{K_2 b d^2 + K_3 A_e (d - d')} = 20.69 \text{ N/mm}^2 \]

\[ f_s = 211.6 \text{ N/mm}^2 \]

\[ \varepsilon_s = 1.058 \times 10^{-3} \]

\[ \varepsilon_{th} = 1.502 \times 10^{-3} \]

\[ ac_1 = 74.5 \text{ mm} \quad \text{at the top corner} \]

\[ W_{cr} = 0.297 \text{ mm} < 0.3 \text{ mm} \quad \text{OK} \]

**Step 27 Design of connections to other elements**

See Chapter 10.

---

**SK 2/58A** Detail of beam at A.

**SK 2/58B** Detail of beam at B.
Example 2.3 Design of beam with torsion

SK 2/59 Two-span edge beam with nib.

Edge beam to carry precast floor slabs on nibs.

Clear gap between beams = 4.5 m
Effective span of beam = 9.0 m

See Example 5.2 for details of precast floor slabs and nib geometry computations.

Two-span beam is fully restrained at the rigid supports.

Step 1 Analysis of beam

Properties of section

SK 2/60 Section of beam with nibs.

Area of section = $500 \times 290 + 2 \times 110 \times 105$

$= 168100 \text{ mm}^2$

Self-weight of beam = $0.1681 \times 24 \text{ kN/m}^3 = 4 \text{ kN/m}$

$\bar{x} = \frac{500 \times 290 \times 145 + 2 \times 110 \times 105 \times (290 + 55)}{168100}$

$= 172.5 \text{ mm}$

$I_{xx} = \frac{1}{12} \times 400 \times 500^3 - \frac{1}{12} \times 110 \times 290^3$

$= 3.943 \times 10^9 \text{ mm}^4$ (gross section)
Assume $\frac{p'}{P} = 0$

Assume $p = 1\%$

Assume $m = \frac{E_x}{E_c} = 10$

From Fig. 11.1,

$F = 6 \times 10^{-2}$

Cracked moment of inertia $= Fbd^3$

$= 6 \times 10^{-2} \times (400 \times 500^3 - 110 \times 290^3)$

$= 2.839 \times 10^9 \text{ mm}^4$

Average moment of inertia, $I_{xx} = 0.5(3.943 + 2.839) \times 10^9 \text{ mm}^4$

$= 3.391 \times 10^9 \text{ mm}^4$

**SK 2/61 Beam geometry to find shear centre $e$.**

Shear centre, $e = \frac{b^2h^2t}{4I_{xx}}$

$b = 400 - 145 = 255 \quad h = 500 - 105 = 395 \quad t = 105$

$e = \frac{255^2 \times 395^2 \times 105}{4 \times 3.391 \times 10^9} = 78.5 \text{ mm}$

**Loading**

Dead load from slab = $5 \text{kN/m}^2 \times 2.25 \text{ m} = 11.25 \text{kN/m}$.

Self-weight of beam = $0.1681 \times 24 \text{kN/m}^3 = 4.0 \text{kN/m}$

Total dead load on beam including self-weight = $15.25 \text{kN/m}$

Live load from slab @ $5 \text{kN/m}^2 = 5 \times 2.25 = 11.25 \text{kN/m}$
Ultimate limit state,

\[ LC_1 = 1.4DL + 1.6LL = 1.4 \times 15.25 + 1.6 \times 11.25 = 22 \text{kN/m} + 18 \text{kN/m} \]

\[ LC_2 = 1.0DL = 15.25 \text{kN/m} \]

Load both spans with \( LC_1 \) to get maximum support moment at B.

Load span AB with \( LC_1 \) and span BC with \( LC_2 \) to get maximum support moment A and maximum span moment at AB.

**Steps 2 and 3** Draw moment and shear envelope
Non-linear analysis with 10% redistribution.

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>Loading</th>
<th>Force</th>
<th>Support</th>
<th>Span</th>
<th>Support</th>
<th>Span</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and C fully restrained</td>
<td>( LC_1 ) on both spans</td>
<td>BM</td>
<td>-270</td>
<td>+135</td>
<td>+270</td>
<td>+135</td>
<td>-270</td>
</tr>
<tr>
<td></td>
<td>Shear</td>
<td>180</td>
<td>-180</td>
<td>180</td>
<td>180</td>
<td>-180</td>
<td>180</td>
</tr>
<tr>
<td>1.4DL on AB</td>
<td>BM</td>
<td>-159.8</td>
<td>+79.9</td>
<td>-125.9</td>
<td>+46.0</td>
<td>-92.0 kNm</td>
<td></td>
</tr>
<tr>
<td>1.0DL on BC</td>
<td>Shear</td>
<td>102.8</td>
<td>95.2</td>
<td>72.6</td>
<td>-65.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( LC_1 ) on AB</td>
<td>BM</td>
<td>-311.7</td>
<td>+155.9</td>
<td>-186.7</td>
<td>+28.0</td>
<td>-70.6 kNm</td>
<td></td>
</tr>
<tr>
<td>( LC_2 ) on BC</td>
<td>Shear</td>
<td>193.9</td>
<td>166.1</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 kN/m LL on AB</td>
<td>BM</td>
<td>-8.44</td>
<td>+4.22</td>
<td>-3.38</td>
<td>-1.0</td>
<td>+1.19 kNm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shear</td>
<td>5.06</td>
<td>3.94</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plastic hinge at A, C fully restrained</td>
<td>1.0 kN/m LL on AB</td>
<td>BM</td>
<td>0</td>
<td>+7.23</td>
<td>-5.78</td>
<td>-1.45</td>
<td>+2.89 kNm</td>
</tr>
<tr>
<td></td>
<td>Shear</td>
<td>3.86</td>
<td>5.14</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assume 10% redistribution. Support moment at A is fixed at 0.9 \( \times 311.7 = 280 \text{kNm} \). The support moment at A reaches 280 kNm elastically with live load on span AB equal to \((280 - 159.8)/8.44 = 14.24 \text{kNm}\). At that point a plastic hinge forms at A and the boundary condition of the structure changes. The remaining live load to go on the span with changed boundary condition is \((18 \text{kN/m} - 14.24 \text{kN/m}) = 3.76 \text{kN/m}\).

Design bending moment at support A = 280 kNm
Design bending moment at midspan AB at centre of span = 79.9 + 14.24 \times 4.22 + 3.76 \times 7.23 = 167.2 kNm
A conservative design span moment = 175 kNm
allowing for the maximum span moment to occur away from the centre of span.
Design bending moment at support B = 270 kNm from elastic analysis \( LC_1 \) on both spans
Design shear at support A = 102.8 + 5.06 \times 14.24 + 3.86 \times 3.76 = 189.5 kN say 190 kN
Design shear at support B = 180 kN \quad (LC_1 \text{ on both spans})

**Step 4** Determine axial loads
Not required.

**Step 5** Determine torsion
Ultimate load from slab = $1.4 \times 11.25 + 1.6 \times 11.25$
\[ = 34 \text{ kN/m} \]

Load assumed to act on edge of nib.

Eccentricity of load from shear centre of beam = $110 - 15$ (chamfer)
\[ + \frac{290}{2} + 78.5 \]
\[ (e = 78.5 \text{ = shear centre}) \]
\[ = 318.5 \text{ mm} \]

Torsion per unit length \[ = 34 \times 0.3185 \]
\[ = 10.83 \text{ kNm/m} \]

Ultimate self-weight of beam \[ = 5.6 \text{ kN/m} \]

Eccentricity of self-weight from shear centre \[ = \bar{x} - \frac{290}{2} + e \]
\[ = 172.5 - 145 + 78.5 \]
\[ = 106 \text{ mm} \]

Torsion per unit length \[ = 5.6 \times 0.106 \]
\[ = 0.59 \text{ kNm/m} \]

Total ultimate torsion in beam \[ = (10.83 + 0.59) \times 4.5 \]
\[ = 51.4 \text{ kNm} \quad \text{at the supports restraining rotation} \]

**Step 6** Cover to reinforcement
Maximum size of aggregate = 20 mm
Maximum size of bar = 25 mm assumed
Maximum size of link = 10 mm
Exposure condition = mild
Fire resistance required = 1 hour
Grade of concrete = C40
Minimum cement content = 325 kg/m$^3$
Maximum free water/cement ratio = 0.55
Nominal cover = 20 mm
Effective depth, $d = 500 - 20 - 10 - 12.5 = 457.5$ mm

**Step 7** Effective span
Effective span = 9.0 m

**Step 8** Effective width of flange
Not required.
Step 9  Slenderness ratio

\[ l = 8.5 \text{ m} = \text{clear span} \]

\[ b_c = 400 \text{ mm} \quad 60b_c = 60 \times 400 = 24000 \text{ mm} > 8500 \text{ mm} \]

\[ d = \left( \frac{M}{0.156 f_{yb} b_c} \right)^{\frac{1}{2}} = \left( \frac{280 \times 10^6}{0.156 \times 40 \times 400} \right)^{\frac{1}{2}} = 335 \text{ mm} \]

\[ \frac{250b_c^2}{d} = \frac{250 \times 400^2}{335} = 119402 \text{ mm} > 8500 \text{ mm} \]

Slenderness check is satisfied.

Step 10  Design for flexure

Support bending moments at A or C = 280 kNm

\[ K = \frac{M}{f_{yb} bd^2} \]

\[ = \frac{280 \times 10^6}{40 \times 400 \times 457.5^2} \]

\[ = 0.0836 < 0.156 \]

No compressive reinforcement required.

\[ z = d \left( 0.5 + \sqrt{\left( 0.25 - \frac{K}{0.9} \right)} \right) \]

\[ = 457.5 \left( 0.5 + \sqrt{\left( 0.25 - \frac{0.0836}{0.9} \right)} \right) \]

\[ = 410 \text{ mm} \]

\[ x = \frac{d - z}{0.45} \]

\[ = \frac{457.5 - 410}{0.45} \]

SK 2/62 Calculation of tensile steel at support.
\[ = 105 \text{ mm} = h_t \]

\[ \therefore \text{Neutral axis is in the flange.} \]

\[ A_s = \frac{M}{0.87f_y z} \]

\[ = \frac{280 \times 10^6}{0.87 \times 460 \times 410} = 1706 \text{ mm}^2 \]

Use 4 no. 25 mm dia. bars (1964 mm²).

Midspan bending moment = 175 kNm

\[ K = \frac{M}{f_{cu} b d^2} \]

\[ = \frac{175 \times 10^6}{40 \times 400 \times 457.5^2} \]

\[ = 0.052 \]

\[ z = 0.94d \]

\[ = 430 \text{ mm} \]

\[ x = \frac{d - z}{0.45} \]

\[ = 61 \text{ mm} < 105 \text{ mm} = h_t \]

\[ A_s = \frac{M}{0.87f_y z} \]

\[ = \frac{175 \times 10^6}{0.87 \times 460 \times 430} \]

\[ = 1017 \text{ mm}^2 \]

Use 2 no. 25 mm dia. bars (982 mm²) + 1 no. 12 mm dia. bar (113 mm²).

**Step 11** Flanged beam

Not required.

**Step 12** Check maximum shear stress at support

\[ v = \frac{V}{bd} \]

\[ = \frac{190 \times 10^3}{290 \times 457.5} \]

\[ = 1.43 \text{ N/mm}^2 \]

\[ 0.8\sqrt{f_{cu}} = 0.8 \times \sqrt{40} = 5 \text{ N/mm}^2 \]

**Step 13** Check flexural shear stress

\[ d = 457.5 \text{ mm} \]

\[ V_A = 190 - 40 \times 0.457 \]

\[ = 172 \text{ kN} \quad \text{at effective depth away from support} \]
\[ 
\nu = \frac{V}{bd} \\
= \frac{172 \times 10^3}{290 \times 457.5} \\
= 1.30 \text{ N/mm}^2 \\
\]

\[ 
P = \frac{100A_s}{bd} \\
= \frac{100 \times 1964}{290 \times 457.5} \\
= 1.48 \\
\]

\[ 
\nu_c = 0.72 \times 1.7 = 0.84 \text{ N/mm}^2 \quad \text{From Fig. 11.5} \\
\]

\[ 
V_{\text{nom}} = (\nu_c + 0.4)bd \\
= (0.84 + 0.4) \times 290 \times 457.5 \times 10^{-3} \\
= 164.5 \text{ kN} \\
\]

\[ 
\nu > \nu_c + 0.4 \\
A_{sv} = \frac{bS_v(\nu - \nu_c)}{0.87f_y} \\
= \frac{290 \times 200 \times (1.30 - 0.84)}{0.87 \times 460} \\
= 66.7 \text{ mm}^2 \quad \text{at 200 mm c/c (2 legs)} \\
\]

\[ 
\frac{A_{sv}}{S_v} = \frac{66.7}{200 \times 2} = 0.17 \quad \text{for each leg} \\
\]

Nominal \[ \frac{A_{sv}}{S_v} = \frac{0.4b}{0.87f_y} \\
= \frac{0.4 \times 290}{0.87 \times 460} \\
= 0.29 \quad \text{(2 legs)} \\
= 0.145 \quad \text{(for each leg)} \\
\]

Area of tension reinforcement required to carry weight of slab on the nib
\[ 
= \frac{34 \text{ kN/m}}{0.87 \times 460} \\
= 85 \text{ mm}^2/\text{m} \\
\]

\[ 
A_s = \frac{85}{1000} = 0.085 \quad \text{for each leg} \\
\]

\textbf{Step 14 Alternative design for shear} \nNot required since design shear is calculated at \( d \) from support.

\textbf{Step 15 Minimum tension reinforcement} \nAssume channel section as L-beam.
\[ A_k > 0.0020b_w h = 0.0020 \times 290 \times 500 = 290 \text{ mm}^2 < 1964 \text{ mm}^2 \text{ provided} \]

**Step 16 Minimum compression reinforcement**
Not required.

**Step 17 Transverse reinforcement in flange**
\[ A_k = 1.5 f_y \text{ mm}^2 / \text{m} \]
\[ = 1.5 \times 105 = 158 \text{ mm}^2 / \text{m} \text{ minimum} \]
(See Example 5.2.)
Reinforcement in nib = 201 mm²/m provided.

**Step 18 Minimum reinforcement in side face of beams**
Not required.

**Step 19 Check deflection**
\[ \frac{b_w}{b} = \frac{290}{400} = 0.725 > 0.3 \]
From Table 11.3,
Basic span/effective depth ratio
for rectangular section = 26 for \( b_w/b = 1.0 \)
for flanged beams = 20.8 for \( b_w/b = 0.3 \)
Interpolated basic ratio = 20.8 + \( \left( \frac{26 - 20.8}{0.7} \right) \times (0.725 - 0.3) = 24 \)
\[ \beta_b = \frac{M}{M'} = \frac{167.2}{155.9} \]
Midspan service stress = \( \left( \frac{5}{8\beta_b} \right) f_y \left( \frac{A_{\text{reqd}}}{A_{\text{prov}}} \right) \)
\[ = \left( \frac{5}{8} \right) \times \left( \frac{155.9}{167.2} \right) \times 460 \times \left( \frac{1017}{1095} \right) \]
\[ = 249 \text{ N/mm}^2 \]
\[ M = 175 \times 10^6 \]
\[ bd^2 = \frac{400 \times 457.5^2}{2} = 2.09 \]
Modification factor for tension reinforcement from Table 11.5 = 1.20
Modified span/depth ratio = 24 \times 1.20 = 28.8
\[ \frac{l_s}{d} = \frac{9000}{457.5} = 19.67 < 28.80 \text{ OK} \]

**Step 20 Maximum area of reinforcement**
\[ A_k < 0.04b_w h = 5800 \text{ mm}^2 \]
Satisfied.
**Step 21** Containment of compression reinforcement
Not required.

**Step 22** Check bearing stress inside bend
Not required.

**Step 23** Curtailment of bars
45 × bar dia. = 45 × 25 = 1125 mm
0.15l = 0.15 × 9000 = 1350 mm
0.25l = 0.25 × 9000 = 2250 mm
2 no. 25 mm dia. top and bottom throughout.
2 no. 25 mm dia. extra top at A, B and C = 5000 long at B, 2500 mm into
span at A and C and properly anchored at A and C.
1 no. 12 mm dia. bottom in spans AB and BC.
Follow simplified detailing rules for beams as in Fig. 2.2.

**Step 24** Spacing of bars
Minimum clear spacing = $MSA + 5 = 20 + 5 = 25$ mm
Actual minimum clear spacing used = 43 mm (support)
Actual maximum clear spacing used = 84 mm (midspan)
Maximum clear spacing allowed = $\frac{47000}{f_s} = \frac{47000}{249} = 189\text{ mm} > 84\text{ mm}$
where $f_s = 249\text{ N/mm}^2$ (see Step 19.)

**Step 25** Check torsional shear stress
Ultimate torsion = 51.4 kN⋅m (see Step 5)
Divide section into 3 rectangles of maximum total torsional stiffness.
*First choice*

![Diagram of beam sections](image)

SK 2/63 Calculation of torsional shear stress.

*Second choice*

SK 2/64 Calculation of torsional shear stress.
500 × 290 – stiffness = \( h_{\text{min}}^2 h_{\text{max}} = 290^3 \times 500 = 1.22 \times 10^{10} \)
2 × 110 × 105 – stiffness = 2 × 105³ × 110 = 0.025 × 10¹⁰
TOTAL = 1.245 × 10¹⁰

Second choice

290 × 290 – stiffness = 290³ × 290 = 0.707 × 10¹⁰
2 × 400 × 105 – stiffness = 2 × 105³ × 400 = 0.0926 × 10¹⁰
TOTAL = 0.7996 × 10¹⁰

Hence the first choice is critical.

Proportion of torsional moment carried by the web = \( \frac{T \times 1.22 \times 10^{10}}{1.245 \times 10^{10}} \)
= \( \frac{51.4 \times 1.22}{1.245} \)
= 50.4 kNm

Torsion carried by flanges = 0.5(51.4 – 50.4) = 0.5 kNm

Torsional shear stress \( \nu_t \) in web = \( \frac{2T}{h_{\text{min}}^2 (h_{\text{max}} - h_{\text{min}}/3)} \)
= \( \frac{2 \times 50.4 \times 10^6}{290^2 \left(290 - \frac{290}{3}\right)} \)
= 2.97 N/mm²

Torsional shear stress \( \nu_t \) in flange = \( \frac{2 \times 0.5 \times 10^6}{105^2 \left(110 - \frac{105}{3}\right)} \)
= 1.21 N/mm²

\( \nu_{t,\text{min}} = 0.067\sqrt{f_{\text{cu}}} \)
= 0.067\sqrt{40} = 0.4 N/mm²

\( \nu_{tu} = 0.8\sqrt{f_{\text{cu}}} = 5 N/mm² \)
\( \nu_{tu} y_1 = \frac{5 \times 450}{550} \)
= 4.1 N/mm²

\( \nu_t > \nu_{t,\text{min}}, \text{torsional reinforcement required.} \)

Torsional shear stress + flexural shear stress = 2.97 + 1.30 \( \text{ (see Step 13) } \)
= 4.27 N/mm²
< 5 N/mm² \( \text{ OK } \)

Torsional reinforcement in web (vertical)
Design of Reinforced Concrete Beams 95

\[ A_{sv} = \frac{T}{S_v \cdot 0.8x_1y_10.87f_y} \]

\[ = \frac{50.4 \times 10^6}{0.8 \times 238 \times 448 \times 0.87 \times 460} \]

\[ = 1.48 \quad \text{(for 2 legs)} \]

\[ = 0.74 \quad \text{(for each leg)} \]

Longitudinal reinforcement for torsion

\[ A_s = \left( \frac{A_{sv}}{S_v} \right) \left( \frac{f_{sy}}{f_y} \right)(x_1 + y_1) \]

\[ = 1.48 \times 1 \times (238 + 448) \]

\[ = 1012 \text{mm}^2 \]

Use 10 no. bars at 101mm\(^2\) each in the longitudinal direction evenly placed on the perimeter of web cross-section \((f_y = 460\text{N/mm}^2)\).

![SK 2/65 A\_sv/S\_v diagram for Example 2.3.](image)

Torsional reinforcement in flange

\[ A_{sv} = \frac{T}{S_v \cdot 0.8x_1y_10.87f_y} \]
Maximum spacing = $x_1$, or $\frac{y_1}{2}$, or 200 mm

= 57 mm

Use 8 mm dia. links at 50 mm centres (1006 mm$^2$/m) ($f_y = 460$ N/mm$^2$)

Could also use 6 mm dia. mild steel links at 50 mm centres (566 mm$^2$/m) ($f_y = 250$ N/mm$^2$).

See Example 5.2, Step 4.

\[ A_{sv} = 131 \text{ mm}^2/\text{m} \quad \text{(460 grade steel)} \]

\[ (\text{for flexure}) = 241 \text{ mm}^2/\text{m} \quad \text{(mild steel Grade 250)} \]

\[ \frac{A_{sv}}{S_v} = 0.078 \quad \text{for torsion (Grade 460)} \]

\[ A_{sv} = 0.078 \times 1000 \times \frac{460}{250} \quad \text{(Grade 250)} \]

= 144 mm$^2$/m (for 2 legs of mild steel)

= 72 mm$^2$/m (for each leg - horizontal)

Total requirement = 241 + 72 = 313 mm$^2$/m < 566 mm$^2$/m

Longitudinal reinforcement for torsion in flange

\[ A_s = \left( \frac{A_{sv}}{S_v} \right) \left( \frac{f_y}{f_y} \right) (x_1 + y_1) \]

\[ = 0.078 \times \frac{460}{250} \times (57 + 352) \]

= 59 mm$^2$ (4 no. 6 mm dia. mild steel: $f_y = 250$ N/mm$^2$)

See Step 10.

At support, $A_s$ required = 1706 mm$^2$

Torsional $A_s$ required at corners (2 bars) = 202 mm$^2$ (Step 25)

Total top reinforcement required = 1706 + 202 = 1908 mm$^2$

Provided = 4 no. 25 mm dia. = 1964 mm$^2$ OK

**Step 26 Flexural crack width calculations**

By elastic analysis: no redistribution.

Maximum support moment at A or C = 201 kNm (serviceability limit state)

\[ d = 500 - 20 - 12 - 12.5 = 455.5 \]

\[ b = 400 \text{ mm} \]

\[ A_s = 1964 \text{ mm}^2 \quad p = 0.0108 \]
SK 2/66 Typical section at support.

\[ A'_s = 982 \text{ mm}^2 \quad p' = 0.0054 \]
\[ m = 10 \]
\[ h_t = 105 \text{ mm} \]
\[ h = 500 \text{ mm} \]
\[ b_t = 290 \text{ mm} \]
\[ d' = 42.5 \text{ mm} \]

\[ x = d \left[ \left( mp + (m - 1)p' \right)^2 + 2 \left( mp + (m - 1) \left( \frac{d'}{d} \right) p' \right) \right]^{1/2} \]

\[ = 156.3 \text{ mm} \]

Using Reference 10, Table 117,

\[ x = \frac{mdA_s + 0.5bh_t^2}{mA_s + bh_t} \]
\[ = \frac{10 \times 455.5 \times 1964 + 0.5 \times 400 \times 105^2}{10 \times 1964 + 400 \times 105} \]
\[ = 181 \text{ mm} \]

\[ z = d - \frac{h_t(3x - 2h_t)}{3(2x - h_t)} \]
\[ = 455.5 - \frac{105(3 \times 181 - 2 \times 105)}{3(2 \times 181 - 105)} \]
\[ = 410 \text{ mm} \]
\[ f_c = \frac{M}{A_z} \]
\[ = \frac{201 \times 10^6}{1964 \times 410} \]
\[ = 250 \text{ N/mm}^2 \]

\[ \varepsilon_c = \frac{f_c}{E_c} = \frac{250}{200 \times 10^3} = 1.25 \times 10^{-3} \]

\[ \varepsilon_h = \frac{(h - x)}{(d - x)} \varepsilon_c \]
\[ = \left( \frac{500 - 181}{455.5 - 181} \right) \times 1.25 \times 10^{-3} \]
\[ = 1.45 \times 10^{-3} \]

\[ \varepsilon_{mh} = \varepsilon_h - \frac{b_t (h - x)^2}{3 E_c A_s (d - x)} \]
\[ = 1.45 \times 10^{-3} - \frac{290 \times (500 - 181)^2}{3 \times 200 \times 10^3 \times 1964 \times (455.5 - 181)} \]
\[ = 1.36 \times 10^{-3} \]

\[ a_{cr} = \sqrt{(44.5^2 + 44.5^2)} - 12.5 = 50.4 \text{ mm} \]

\[ W_{cr} = \frac{3 a_{cr} \varepsilon_{mh}}{1 + \frac{2(a_{cr} - \varepsilon_{min})}{(h - x)}} \]
\[ = \frac{3 \times 50.4 \times 1.36 \times 10^{-3}}{1 + \frac{2(50.4 - 32)}{(500 - 181)}} \]
\[ = 0.18 \text{ mm} < 0.3 \text{ mm} \]

*Step 27: Design of connections to other components*

Follow Chapter 10.
2.5 FIGURES FOR CHAPTER 2

Fig. 2.1 Values of $\beta_F$.
Fig. 2.2 Simplified detailing rules for beams.